OCR
Oxford Cambridge and RSA

Friday 7 June 2024 – Afternoon

AS Level Further Mathematics A

Y533/01 Mechanics

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 A particle P of mass 2.5 kg is moving with a constant speed of $4\,\mathrm{m\,s^{-1}}$ in a straight line on a smooth horizontal plane when it collides directly with a fixed vertical wall.

After the collision P moves away from the wall with a speed of $2.8\,\mathrm{m\,s^{-1}}$.

(a) Calculate the coefficient of restitution between P and the wall. **[1]**

(b) Find the magnitude and state the direction of the impulse exerted on P by the wall. **[3]**

(c) State the magnitude and direction of the impulse exerted on the wall by P. **[2]**

2 A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 1.8 m. The other end of the string is attached to a fixed point, O, on a smooth horizontal plane. Initially, P is moving with a constant speed of $12\,\mathrm{m\,s^{-1}}$ in a horizontal circle with O as its centre.

(a) **(i)** Find the magnitude of the acceleration of P. **[1]**

(ii) State the direction of the acceleration of P. **[1]**

A force is now applied to P in such a way that its angular velocity increases. At the instant that the angular velocity reaches $8\,\mathrm{rad\,s^{-1}}$, the string breaks.

(b) **(i)** Find the speed with which P is moving at the instant that the string breaks. **[1]**

(ii) Find the tension in the string at the instant that the string breaks. **[2]**

After the string has broken P starts to move directly up a smooth slope which is fixed to the plane and inclined at an angle $\theta°$ above the horizontal. Particle P moves a distance of 20 m up the slope before coming to instantaneous rest.

(c) Use an energy method to determine the value of θ. **[3]**

3 A small object P of mass m is suspended from a fixed point by a light inextensible string of length l. When P is displaced and released in a certain way, it oscillates in a vertical plane. The time taken for one complete oscillation is called the period and is denoted by τ.

A student is carrying out experiments with P and suggests the following formula to model the value of τ.

$\tau = cg^{\alpha}l^{\beta}m^{\gamma}$

in which

- g is the acceleration due to gravity,
- c is a dimensionless constant.

(a) Use dimensional analysis to determine the values of the constants α, β and γ. **[5]**

(b) (i) Determine the effect on the period, according to the model, if the length of the string is then multiplied by 4, all other conditions being unchanged. **[2]**

(ii) Determine the effect on the period, according to the model, if **instead** the mass of the object is multiplied by 4, all other conditions being unchanged. **[1]**

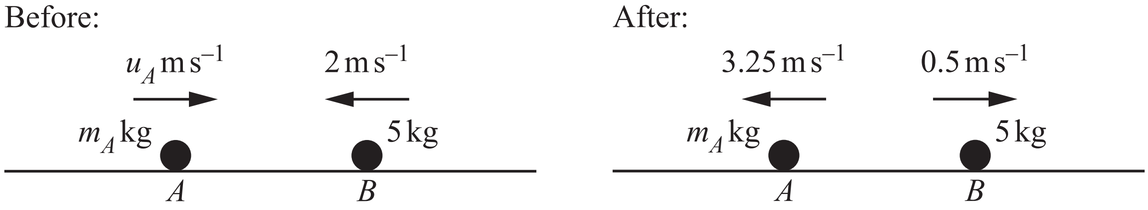
4 A particle B of mass 5 kg is at rest at the bottom of a slope which is angled at $\sin^{-1} 0.2$ above the horizontal. A constant force D initially acts directly up the slope on B.

The total resistance to the motion of B is modelled as being a constant 12 N.

At the instant that D stops acting, the speed of B is $18\,\text{m}\,\text{s}^{-1}$ and B has moved 90 m up the slope.

Determine the average power of D over the time that D has been acting on B. **[6]**

5 Two particles, A of mass m_A kg and B of mass 5 kg, are moving directly towards each other on a smooth horizontal floor. Before they collide they have speeds $u_A \, \text{m s}^{-1}$ and $2 \, \text{m s}^{-1}$ respectively. Immediately after they collide the direction of motion of each particle has been reversed and A and B have speeds $3.25 \, \text{m s}^{-1}$ and $0.5 \, \text{m s}^{-1}$ respectively (see diagram). The coefficient of restitution between A and B is 0.75.



(a) Determine the value of m_A and the value of u_A. **[5]**

(b) Show that approximately 41% of the kinetic energy of the system is lost in this collision. **[3]**

After the collision between A and B, B goes on to collide directly with a third particle C of mass 3 kg which is travelling towards B with a speed of $5.5 \, \text{m s}^{-1}$. The coefficient of restitution between B and C is denoted by e.

(c) Given that, after B and C collide, there are no further collisions between A, B and C determine the range of possible values of e. **[6]**

6 A motorbike and its rider, together denoted by M, have a combined mass of 360 kg. The resistive force experienced by M when it is in motion is modelled as being proportional to the speed it is moving at. All motion of M is on a straight horizontal road.

It is found that with the engine of the motorbike working at a rate of 12 kW, the maximum constant speed that M can move at is $10 \, \text{m s}^{-1}$.

Determine the speed of M such that with the engine working at a rate of 12 kW the acceleration of M is $1.5 \, \text{m s}^{-2}$. **[7]**

7 A particle P of mass 3.5 kg is attached to one end of a rod of length 5.4 m. The other end of the rod is hinged at a fixed point O and P hangs in equilibrium directly below O.

A horizontal impulse of magnitude 44.1 N s is applied to P.

In an initial model of the subsequent motion of P the rod is modelled as being light and inextensible and all resistance to the motion of P is ignored. You are given that P moves in a circular path in a vertical plane containing O. The angle that the rod makes with the downward vertical through O is θ radians.

(a) Determine the largest value of θ in the subsequent motion of P. **[5]**

In a **revised** model the rod is still modelled as being light and inextensible but the resistance to the motion of P is not ignored. Instead, it is modelled as causing a loss of energy of 20 J for every metre that P travels.

(b) Show that according to the **revised** model, the maximum value of θ in the subsequent motion of P satisfies the following equation.

$$343\left(1+2\cos\theta\right)=400\theta$$ **[2]**

You are given that $\theta = 1.306$ is the solution to the above equation, correct to **4** significant figures.

(c) Determine the difference in the predicted maximum vertical heights attained by P using the two models. Give your answer correct to **3** significant figures. **[3]**

(d) Suggest **one** further improvement that could be made to the model of the motion of P. **[1]**

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