

# Monday 13 May 2024 – Afternoon

## **AS Level Further Mathematics A**

## Y531/01 Pure Core

#### Time allowed: 1 hour 15 minutes

#### You must have:

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- · a scientific or graphical calculator

#### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer** Booklet. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

56 338564 388564

64 338564 338564 338564 64 338564 338564 338564 64 338564 338564 338564

 $\begin{array}{c} 64 \\ 338564 \\ 64 \\ 3385$ 

64 338564 338564 -

64 338564 338564 3 64 338564 338564 338 64 338564 338564 338<sup>5</sup>

64 338564 338564 3 64 338564 338564 338 64 338564 338564 338 64 338564 338564 338

64 338564 338564 338 64 338564 338564 338564 33856

 $\begin{array}{c} 64 \ 3385_{64} \ 3385_{$ 

 $\begin{array}{c} 338564 \\ 33856$ 

<sup>3</sup>38564

3<sup>38564</sup> 338564 <sup>338564</sup> 338564 38564 338564 <sup>338564</sup>

3564 338564 <sup>338564</sup> 3564 338564 <sup>338564</sup>

3564 338564 <sup>33</sup>8564 3564 338564 <sup>33</sup>8564

3564 338564 <sup>338</sup>564 3564 338564 <sup>338</sup>564

3564 338564 <sup>33</sup>8564 3564 338564 <sup>33</sup>8564

3564 338564 <sup>338</sup>564 3564 338564 <sup>338</sup>564

3564 338564 <sup>33</sup>8564 3564 338564 <sup>33</sup>8564

3564 338564 <sup>338</sup>564 3564 338564 <sup>338</sup>564

8564 338564 338564 338564 338564 338564 338564 338564

<sup>338564</sup> 338564 <sup>338564</sup> <sup>338564</sup> <sup>338564</sup> <sup>338564</sup> <sup>338564</sup>

- Fill in the boxes on the front of the Printed Answer Booklet. •
- Answer all the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- · Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $gm s^{-2}$ . When a numerical value is needed use g = 9.8 unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

#### **INFORMATION**

- The total mark for this paper is 60.
- The marks for each question are shown in brackets [].
- This document has 8 pages.

### ADVICE

Read each question carefully before you start your answer.



- 1 Use a matrix method to determine the solution of the following simultaneous equations. [4]
  - 2x-3y+z = 1 x-2y-4z = 405x+6y-z = 61

#### 2 In this question you must show detailed reasoning.

- (a) Express  $\frac{8+i}{2-i}$  in the form a+bi where a and b are real. [2]
- (b) Solve the equation  $4x^2 8x + 5 = 0$ . Give your answer(s) in the form c + di where c and d are real. [2]

**3** (a) (i) Find 
$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$$
. [1]

(ii) State a geometrical relationship between the answer to part (a)(i) and the

vectors 
$$\begin{pmatrix} 1\\2\\-1 \end{pmatrix}$$
 and  $\begin{pmatrix} 3\\5\\-2 \end{pmatrix}$ . [1]

- (iii) Verify the relationship stated in part (a)(ii). [2]
- (b) Find the angle between the vectors 2i-2j+k and 4i-j+8k. [3]

4 The Argand diagram shows a circle of radius 3. The centre of the circle is the point which represents the complex number 4-2i.



(a) Use set notation to define the locus of complex numbers, z, represented by points which lie on the circle. [2]

The locus *L* is defined by  $L = \{z : z \in \mathbb{C}, |z-i| = |z+2|\}.$ 

(b) On the Argand diagram in the Printed Answer Booklet, sketch and label the locus L. [2]

You are given that the locus  $\{z : z \in \mathbb{C}, \arg(z-1) = \frac{1}{4}\pi, \operatorname{Re}(z) = 3\}$  contains only one number.

- (c) Find this number.
- 5 The line through points A(8, -7, -2) and B(11, -9, 0) is denoted by  $L_1$ .

(a) Find a vector equation for 
$$L_1$$
. [2]

(b) Determine whether the point (26, -19, -14) lies on  $L_1$ . [2]

The line  $L_2$  passes through the origin, O, and intersects  $L_1$  at the point C. The lines  $L_1$  and  $L_2$  are perpendicular.

- (c) By using the fact that C lies on  $L_1$ , find a vector equation for  $L_2$ . [4]
- (d) Hence find the shortest distance from O to  $L_1$ . [2]

6 You are given that 
$$\mathbf{A} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$$
 where *a* is a constant.  
Prove by induction that  $\mathbf{A}^n = \begin{pmatrix} 1 & an \\ 0 & 1 \end{pmatrix}$  for all integers  $n \ge 1$ . [5]

[2]

#### 7 In this question you must show detailed reasoning.

The roots of the equation  $2x^3 - 3x^2 - 3x + 5 = 0$  are  $\alpha$ ,  $\beta$  and  $\gamma$ . By considering  $(\alpha + \beta + \gamma)^2$  and  $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$ , determine a cubic equation with integer coefficients whose roots are  $\frac{\alpha\beta}{\gamma}$ ,  $\frac{\beta\gamma}{\alpha}$  and  $\frac{\gamma\alpha}{\beta}$ . [6]

8 Three transformations,  $T_A$ ,  $T_B$  and  $T_C$ , are represented by the matrices A, B and C respectively.

You are given that  $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ .

- (a) Find the matrix which represents the inverse transformation of  $T_A$ . [1]
- (b) By considering matrix multiplication, determine whether T<sub>A</sub> followed by T<sub>B</sub> is the same transformation as T<sub>B</sub> followed by T<sub>A</sub>.
   [2]

Transformations R and S are each defined as being the result of successive transformations, as specified in the table.

Transformation	First transformation	followed by	
R	$T_A$ followed by $T_B$	T <sub>C</sub>	
S	T <sub>A</sub>	$T_B$ followed by $T_C$	

(c) Explain, using a property of matrix multiplication, why R and S are the same transformations. [2]

A quadrilateral, Q, has vertices D, E, F and G in anticlockwise order from D. Under transformation R, Q's image, Q', has vertices D', E', F' and G' (where D' is the image of D, etc). The area of Q, in suitable units, is 5.

You are given that det  $C = a^2 + 1$  where *a* is a real constant.

(	d)	<i>(</i> i)	Determine the order of the vertices of $O'$ starting anticlockwise from $D'$	[2]
U	u)	(1)	) Determine the order of the vertices of $\mathcal{Q}$ , starting anticidekwise from $D$ .	4

- (ii) Find, in terms of a, the area of Q'.
- (iii) Explain whether the inverse transformation for R exists. Justify your answer. [2]

[1]

## 9 In this question you must show detailed reasoning.

You are given that *a* is a real root of the equation  $x^4 + x^3 + 3x^2 - 5x = 0$ .

You are also given that a + 2 + 3i is one root of the equation

$$z^{4} - 2(1+a)z^{3} + (21a - 10)z^{2} + (86 - 80a)z + (285a - 195) = 0.$$

Determine all possible values of *z*.

## **END OF QUESTION PAPER**

[8]

## **BLANK PAGE**

## **BLANK PAGE**



#### **Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

8

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.