OCR
Oxford Cambridge and RSA

Monday 13 May 2024 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 Use a matrix method to determine the solution of the following simultaneous equations. **[4]**

$$\begin{aligned} 2x-3y+\ z &= \ 1 \\ x-2y-4z &= 40 \\ 5x+6y-\ z &= 61 \end{aligned}$$

2 **In this question you must show detailed reasoning.**

(a) Express $\frac{8+\mathrm{i}}{2-\mathrm{i}}$ in the form $a+b\mathrm{i}$ where a and b are real. **[2]**

(b) Solve the equation $4x^2-8x+5=0$. Give your answer(s) in the form $c+d\mathrm{i}$ where c and d are real. **[2]**

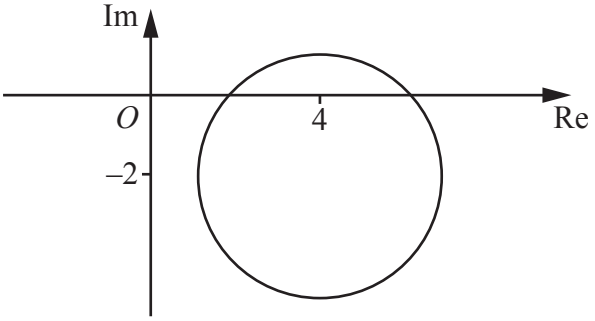
3 **(a)** **(i)** Find $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$. **[1]**

(ii) State a geometrical relationship between the answer to part **(a)(i)** and the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$. **[1]**

(iii) Verify the relationship stated in part **(a)(ii)**. **[2]**

(b) Find the angle between the vectors $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and $4\mathbf{i}-\mathbf{j}+8\mathbf{k}$. **[3]**

4 The Argand diagram shows a circle of radius 3. The centre of the circle is the point which represents the complex number $4-2\mathrm{i}$.



(a) Use set notation to define the locus of complex numbers, z, represented by points which lie on the circle. **[2]**

The locus L is defined by $L = \{z : z \in \mathbb{C}, |z-\mathrm{i}| = |z+2|\}$.

(b) On the Argand diagram in the Printed Answer Booklet, sketch and label the locus L. **[2]**

You are given that the locus $\left\{z : z \in \mathbb{C}, \arg(z-1) = \frac{1}{4}\pi, \mathrm{Re}(z) = 3\right\}$ contains only one number.

(c) Find this number. **[2]**

5 The line through points $A(8, -7, -2)$ and $B(11, -9, 0)$ is denoted by L_1.

(a) Find a vector equation for L_1. **[2]**

(b) Determine whether the point $(26, -19, -14)$ lies on L_1. **[2]**

The line L_2 passes through the origin, O, and intersects L_1 at the point C. The lines L_1 and L_2 are perpendicular.

(c) By using the fact that C lies on L_1, find a vector equation for L_2. **[4]**

(d) Hence find the shortest distance from O to L_1. **[2]**

6 You are given that $\mathbf{A} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ where a is a constant.

Prove by induction that $\mathbf{A}^n = \begin{pmatrix} 1 & an \\ 0 & 1 \end{pmatrix}$ for all integers $n \geqslant 1$. **[5]**

7 **In this question you must show detailed reasoning.**

The roots of the equation $2x^3 - 3x^2 - 3x + 5 = 0$ are α, β and γ.

By considering $(\alpha+\beta+\gamma)^2$ and $(\alpha\beta+\beta\gamma+\gamma\alpha)^2$, determine a cubic equation with integer coefficients whose roots are $\frac{\alpha\beta}{\gamma}$, $\frac{\beta\gamma}{\alpha}$ and $\frac{\gamma\alpha}{\beta}$. **[6]**

8 Three transformations, T_A, T_B and T_C, are represented by the matrices **A**, **B** and **C** respectively.

You are given that $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(a) Find the matrix which represents the inverse transformation of T_A. **[1]**

(b) By considering matrix multiplication, determine whether T_A followed by T_B is the same transformation as T_B followed by T_A. **[2]**

Transformations R and S are each defined as being the result of successive transformations, as specified in the table.

Transformation	First transformation	followed by
R	T_A followed by T_B	T_C
S	T_A	T_B followed by T_C

(c) Explain, using a property of matrix multiplication, why R and S are the same transformations. **[2]**

A quadrilateral, Q, has vertices D, E, F and G in anticlockwise order from D. Under transformation R, Q's image, Q', has vertices D', E', F' and G' (where D' is the image of D, etc). The area of Q, in suitable units, is 5.

You are given that $\det \mathbf{C} = a^2 + 1$ where a is a real constant.

(d) **(i)** Determine the order of the vertices of Q', starting anticlockwise from D'. **[2]**

(ii) Find, in terms of a, the area of Q'. **[1]**

(iii) Explain whether the inverse transformation for R exists. Justify your answer. **[2]**

9 In this question you must show detailed reasoning.

You are given that a is a real root of the equation $x^4 + x^3 + 3x^2 - 5x = 0$.

You are also given that $a + 2 + 3\mathrm{i}$ is one root of the equation

$$z^4 - 2(1+a)z^3 + (21a - 10)z^2 + (86 - 80a)z + (285a - 195) = 0.$$

Determine all possible values of z. **[8]**

END OF QUESTION PAPER

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