

Thursday 22 October 2020 – Afternoon

A Level Further Mathematics A

Y544/01 Discrete Mathematics

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

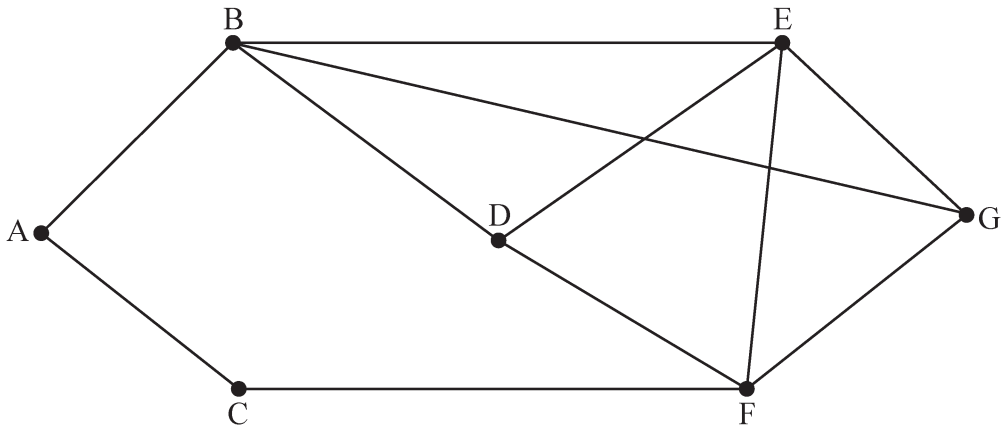
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

1 This question is about the planar graph shown below.



- (a) (i) Apply Kuratowski's theorem to verify that the graph is planar. [2]
- (ii) Use Euler's formula to calculate the number of regions in a planar representation of the graph. [1]
- (b) (i) Write down a Hamiltonian cycle for the graph. [1]
- (ii) By finding a suitable pair of vertices, show that Ore's theorem cannot be used to prove that the graph, as shown above, is Hamiltonian. [2]
- (c) (i) Draw the graph formed by using the contractions AB and CF. [1]
- (ii) Use Ore's theorem to show that this contracted graph is Hamiltonian. [2]

- 2 Annie and Brett play a two-person, simultaneous play game. The table shows the pay-offs for Annie and Brett in the form (a, b) . So, for example, if Annie plays strategy K and Brett plays strategy S, Annie wins 2 points and Brett wins 6 points.

		Brett		
		R	S	T
Annie	K	(7, 3)	(2, 6)	(5, 3)
	L	(1, 5)	(8, 2)	(2, 5)
	M	(3, 2)	(1, 5)	(4, 6)

- (a) (i) Determine the play-safe strategy for Annie. [2]
- (ii) Show that the play-safe strategy for Brett is T. [2]
- (b) (i) If Annie knows that Brett is planning on playing strategy T, which strategy should Annie play to maximise her points? [1]
- (ii) If Brett knows that Annie is planning on playing the strategy identified in part (b)(i), which strategy should Brett play to maximise his points? [1]
- (c) Show that, for Brett, strategy R is weakly dominated. [2]
- (d) Using a graphical method, determine the optimal mixed strategy for Brett. [4]
- (e) Show that the game has no Nash equilibrium points. [2]

3 An initial simplex tableau is shown below.

P	x	y	z	s	t	RHS
1	-3	1	0	0	0	0
0	2	0	1	1	0	18
0	-1	2	3	0	1	20

(a) Write down the objective for the problem that is represented by this initial tableau. [2]

Variables s and t are slack variables.

(b) Use the final row of the initial tableau to explain what a slack variable is. [2]

(c) Carry out one iteration of the simplex algorithm and hence:

- give the pivot column used and the value of the pivot element
- write down the value of P after this iteration
- find the values of x , y and z after this iteration
- describe the effect of the iteration geometrically. [8]

- 4 (a) Show that there are 127 ways to partition a set of 8 distinct elements into two non-empty subsets. [3]

A group of 8 people (A, B, ...) have 8 reserved seats (1, 2, ...) on a coach. Seat 1 is reserved for person A, seat 2 for person B, and so on. The reserved seats are labelled but the individual people do not know which seat has been reserved for them.

The first 4 people, A, B, C and D, choose their seats at random from the 8 reserved seats.

- (b) Determine how many different arrangements there are for the seats chosen by A, B, C and D. [2]

The group organiser moves A, B, C and D to their correct seats (A in seat 1, B in seat 2, C in seat 3 and D in seat 4).

The other 4 people (E, F, G and H) then choose their seats at random from the remaining 4 reserved seats (5, 6, 7 and 8).

- (c) List the 9 derangements of {E, F, G, H}, where none of these four people is in the seat that has been reserved for them. [3]

Suppose, instead, that the 8 people had chosen their seats at random from the 8 reserved seats, without the organiser intervening.

- (d) Determine the total number of ways in which the seats can be chosen so that 4 of the people are in their correct seats and 4 are not in their correct seats. [2]

- 5 The manager of a farm shop wants to pave routes on the farm so that, after visiting the shop, customers can visit the animals in fields A, B, C, D and E.

The table shows the cost, in £, of making a paved path between each pair of fields.

A river means that it is not possible to make a paved path between C and E.

A, B	A, C	A, D	A, E	B, C	B, D	B, E	C, D	C, E	D, E
300	500	900	700	200	600	400	500	–	100

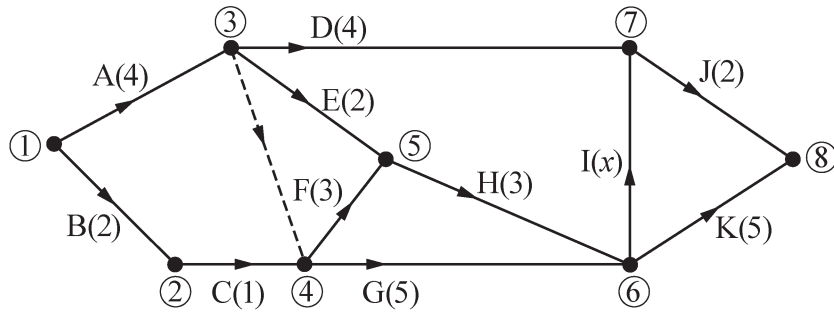
- (a) Determine the minimum cost of connecting the fields. [2]
- (b) (i) By applying the lower bound algorithm to each vertex in turn, determine a best lower bound for P , the minimum cost of making a circular tour (cycle) of paved paths that visits each field once. [4]
- (ii) By applying the nearest neighbour algorithm, starting at each vertex in turn, find a best upper bound for P . You do not need to attempt any route improvements. [4]
- (iii) Give the order in which the fields are visited in a circular tour of paved paths that corresponds to the best upper bound found in part (b)(ii). [2]
- (c) Give a practical reason why the total cost of paving for the project might be more than the best upper bound found in part (b)(ii). [1]

It becomes possible to use an existing bridge to make a paved route between C and E.

Using this bridge, there is a new indirect route from A to D that costs less than £900 to pave.

- (d) When this bridge is used, what can be determined about the minimum cost of
- (i) paving the route between C and E [2]
- (ii) connecting all the fields? [2]

- 6 A project is represented by the activity on arc network below.



The duration of each activity (in minutes) is shown in brackets, apart from activity I.

- (a) Suppose that the minimum completion time for the project is 15 minutes.
- (i) By calculating the early event times, determine the range of values for x . [3]
- (ii) By calculating the late event times, determine which activities **must** be critical. [3]

The table shows the number of workers needed for each activity.

Activity	A	B	C	D	E	F	G	H	I	J	K
Workers	2	1	1	2	n	1	2	1	1	1	4

- (b) Determine the maximum possible value for n if 5 workers can complete the project in 15 minutes. Explain your reasoning. [4]

The duration of activity F is reduced to 1.5 minutes, but only 4 workers are available. The minimum completion time is no longer 15 minutes.

- (c) Determine the minimum project completion time in this situation. [1]
- (d) Find the maximum possible value for x for this minimum project completion time. [1]
- (e) Find the maximum possible value for n for this minimum project completion time. [1]

END OF QUESTION PAPER

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