Oxford Cambridge and RSA

# Tuesday 18 June 2019 – Morning

## A Level Further Mathematics A

### Y543/01 Mechanics

**Time allowed: 1 hour 30 minutes**

**You must have:**
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

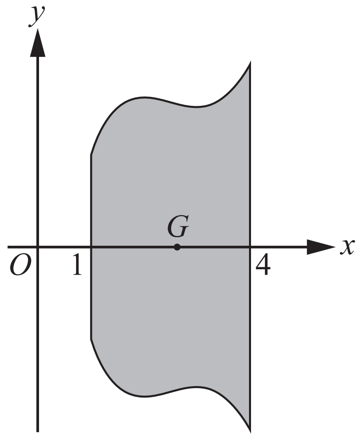
**You may use:**
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

**INFORMATION**
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

**1** The region bounded by the $x$-axis, the curve $y=\sqrt{2x^3-15x^2+36x-20}$ and the lines $x=1$ and $x=4$ is rotated through $2\pi$ radians about the $x$-axis to form a uniform solid of revolution $R$. The centre of mass of $R$ is the point $G$ (see diagram).



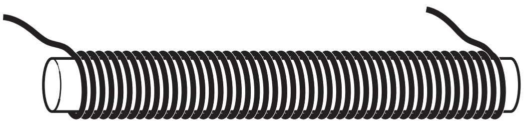

**(a)** Explain why the $y$-coordinate of $G$ is 0. **[1]**

**(b)** Find the $x$-coordinate of $G$. **[4]**

$P$ is a point on the edge of the curved surface of $R$ where $x=4$. $R$ is freely suspended from $P$ and hangs in equilibrium.

**(c)** Find the angle between the axis of symmetry of $R$ and the vertical. **[3]**

**2** A solenoid is a device formed by winding a wire tightly around a hollow cylinder so that the wire forms (approximately) circular loops along the cylinder (see diagram).



When the wire carries an electrical current a magnetic field is created inside the solenoid which can cause a particle which is moving inside the solenoid to accelerate.

A student is carrying out experiments on particles moving inside solenoids. His professor suggests that, for a particle of mass $m$ moving with speed $v$ inside a solenoid of length $h$, the acceleration $a$ of the particle can be modelled by a relationship of the form $a=km^{\alpha}v^{\beta}h^{\gamma}$, where $k$ is a constant. The professor tells the student that $[k]=\mathrm{MLT}^{-1}$.

**(a)** Use dimensional analysis to find $\alpha$, $\beta$ and $\gamma$. **[6]**

**(b)** The mass of an electron is $9.11\times10^{-31}$ kg and the mass of a proton is $1.67\times10^{-27}$ kg.

For an electron and a proton moving inside the same solenoid with the same speed, use the model to find the ratio of the acceleration of the electron to the acceleration of the proton. **[3]**

**(c)** The professor tells the student that $a$ also depends on the number of turns or loops of wire, $N$, that the solenoid has.

Explain why dimensional analysis **cannot** be used to determine the dependence of $a$ on $N$. **[1]**

**3** A particle $Q$ of mass $m$ kg is acted on by a single force so that it moves with constant acceleration $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}\mathrm{m\,s^{-2}}$. Initially $Q$ is at the point $O$ and is moving with velocity $\mathbf{u} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}\mathrm{m\,s^{-1}}$.

After $Q$ has been moving for 5 seconds it reaches the point $A$.

**(a)** Use the equation $\mathbf{v.v} = \mathbf{u.u} + 2\mathbf{a.x}$ to show that at $A$ the kinetic energy of $Q$ is $37m$ J. **[5]**

**(b)** **(i)** Show that the power initially generated by the force is $-8m$ W. **[2]**

**(ii)** The power in part **(b)(i)** is negative. Explain what this means about the initial motion of $Q$. **[1]**

**(c)** **(i)** Find the time at which the power generated by the force is instantaneously zero. **[3]**

**(ii)** Find the minimum kinetic energy of $Q$ in terms of $m$. **[2]**

**4** A right circular cone $C$ of height 4 m and base radius 3 m has its base fixed to a horizontal plane. One end of a light elastic string of natural length 2 m and modulus of elasticity 32 N is fixed to the vertex of $C$. The other end of the string is attached to a particle $P$ of mass 2.5 kg.

$P$ moves in a horizontal circle with constant speed and in contact with the smooth curved surface of $C$. The extension of the string is 1.5 m.

**(a)** Find the tension in the string. **[2]**

**(b)** Find the speed of $P$. **[7]**

**5** A particle $P$ of mass 4.5 kg is free to move along the $x$-axis. In a model of the motion it is assumed that $P$ is acted on by two forces:

- a constant force of magnitude $f$ N in the positive $x$ direction;
- a resistance to motion, $R$ N, whose magnitude is proportional to the speed of $P$.

At time $t$ seconds the velocity of $P$ is $v\,\mathrm{m\,s^{-1}}$. When $t = 0$, $P$ is at the origin $O$ and is moving in the positive direction with speed $u\,\mathrm{m\,s^{-1}}$, and when $v = 5$, $R = 2$.

**(a)** Show that, according to the model, $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{10f - 4v}{45}$. **[2]**

**(b)** **(i)** By solving the differential equation in part **(a)**, show that $v = \frac{1}{2}\left(5f - (5f - 2u)\mathrm{e}^{-\frac{4}{45}t}\right)$. **[5]**

**(ii)** Describe briefly how, according to the model, the speed of $P$ varies over time in each of the following cases.

- $u < 2.5f$
- $u = 2.5f$
- $u > 2.5f$

**[3]**

**(c)** In the case where $u = 2f$, find in terms of $f$ the exact displacement of $P$ from $O$ when $t = 9$. **[4]**

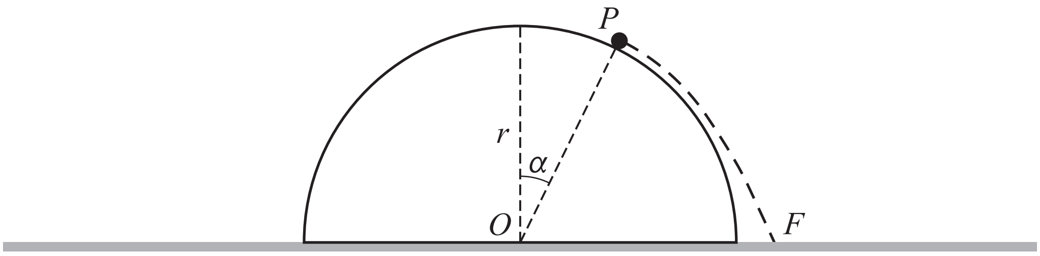
**6** Two particles $A$ and $B$, of masses $m$ kg and 1 kg respectively, are connected by a light inextensible string of length $d$ m and placed at rest on a smooth horizontal plane a distance of $\frac{1}{2}d$ m apart. $B$ is then projected horizontally with speed $v$ m s$^{-1}$ in a direction perpendicular to $AB$.

**(a)** Show that, at the instant that the string becomes taut, the magnitude of the instantaneous impulse in the string, $I$ N s, is given by $I = \frac{\sqrt{3}mv}{2(1+m)}$. **[4]**

**(b)** Find, in terms of $m$ and $v$, the kinetic energy of $B$ at the instant after the string becomes taut. Give your answer as a single algebraic fraction. **[3]**

**(c)** In the case where $m$ is very large, describe, with justification, the approximate motion of $B$ after the string becomes taut. **[2]**

**7**




The flat surface of a smooth solid hemisphere of radius $r$ is fixed to a horizontal plane on a planet where the acceleration due to gravity is denoted by $\gamma$. $O$ is the centre of the flat surface of the hemisphere.

A particle $P$ is held at a point on the surface of the hemisphere such that the angle between $OP$ and the upward vertical through $O$ is $\alpha$, where $\cos\alpha = \frac{3}{4}$.

$P$ is then released from rest. $F$ is the point on the plane where $P$ first hits the plane (see diagram).

**(a)** Find an exact expression for the distance $OF$. **[11]**

The acceleration due to gravity on and near the surface of the planet Earth is roughly $6\gamma$.

**(b)** Explain whether $OF$ would increase, decrease or remain unchanged if the action were repeated on the planet Earth. **[1]**

**END OF QUESTION PAPER**