



Oxford Cambridge and RSA

**Monday 13 May 2019 – Afternoon**

**AS Level Further Mathematics A**

**Y531/01 Pure Core**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g\text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 You are given that  $z = 3 - 4i$ .

(a) Find

- $|z|$ ,
- $\arg(z)$ ,
- $z^*$ .

[3]

On an Argand diagram the complex number  $w$  is represented by the point  $A$  and  $w^*$  is represented by the point  $B$ .

(b) Describe the geometrical relationship between the points  $A$  and  $B$ .

[2]

2 Matrices  $\mathbf{P}$  and  $\mathbf{Q}$  are given by  $\mathbf{P} = \begin{pmatrix} 1 & k & 0 \\ -2 & 1 & 3 \end{pmatrix}$  and  $\mathbf{Q} = \begin{pmatrix} (1+k) & -1 \end{pmatrix}$  where  $k$  is a constant.

Exactly one of statements A and B is true.

Statement A:  $\mathbf{P}$  and  $\mathbf{Q}$  (in that order) are conformable for multiplication.

Statement B:  $\mathbf{Q}$  and  $\mathbf{P}$  (in that order) are conformable for multiplication.

(a) State, with a reason, which **one** of A and B is true.

[2]

(b) Find either  $\mathbf{PQ}$  or  $\mathbf{QP}$  in terms of  $k$ .

[2]

3 The position vector of point  $A$  is  $\mathbf{a} = -9\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ .

The line  $l$  passes through  $A$  and is perpendicular to  $\mathbf{a}$ .

(a) Determine the shortest distance between the origin,  $O$ , and  $l$ .

[2]

$l$  is also perpendicular to the vector  $\mathbf{b}$  where  $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$ .

(b) Find a vector which is perpendicular to both  $\mathbf{a}$  and  $\mathbf{b}$ .

[1]

(c) Write down an equation of  $l$  in vector form.

[1]

$P$  is a point on  $l$  such that  $PA = 2OA$ .

(d) Find angle  $POA$  giving your answer to 3 significant figures.

[3]

$C$  is a point whose position vector,  $\mathbf{c}$ , is given by  $\mathbf{c} = p\mathbf{a}$  for some constant  $p$ . The line  $m$  passes through  $C$  and has equation  $\mathbf{r} = \mathbf{c} + \mu\mathbf{b}$ . The point with position vector  $9\mathbf{i} + 8\mathbf{j} - 12\mathbf{k}$  lies on  $m$ .

(e) Find the value of  $p$ .

[3]

**4 In this question you must show detailed reasoning.**

You are given that  $f(z) = 4z^4 - 12z^3 + 41z^2 - 128z + 185$  and that  $2 + i$  is a root of the equation  $f(z) = 0$ .

(a) Express  $f(z)$  as the product of two quadratic factors with integer coefficients. [5]

(b) Solve  $f(z) = 0$ . [3]

Two loci on an Argand diagram are defined by  $C_1 = \{z: |z| = r_1\}$  and  $C_2 = \{z: |z| = r_2\}$  where  $r_1 > r_2$ . You are given that two of the points representing the roots of  $f(z) = 0$  are on  $C_1$  and two are on  $C_2$ .  $R$  is the region on the Argand diagram between  $C_1$  and  $C_2$ .

(c) Find the exact area of  $R$ . [4]

(d)  $\omega$  is the sum of all the roots of  $f(z) = 0$ .

Determine whether or not the point on the Argand diagram which represents  $\omega$  lies in  $R$ . [2]

**5 In this question you must show detailed reasoning.**

You are given that  $\alpha$ ,  $\beta$  and  $\gamma$  are the roots of the equation  $5x^3 - 2x^2 + 3x + 1 = 0$ .

(a) Find the value of  $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ . [5]

(b) Find a cubic equation whose roots are  $\alpha^2$ ,  $\beta^2$  and  $\gamma^2$  giving your answer in the form  $ax^3 + bx^2 + cx + d = 0$  where  $a$ ,  $b$ ,  $c$  and  $d$  are integers. [4]

**6** A transformation  $T$  is represented by the matrix  $\mathbf{T}$  where  $\mathbf{T} = \begin{pmatrix} x^2 + 1 & -4 \\ 3 - 2x^2 & x^2 + 5 \end{pmatrix}$ .

A quadrilateral  $Q$ , whose area is 12 units, is transformed by  $T$  to  $Q'$ .

Find the smallest possible value of the area of  $Q'$ . [5]

**Turn over for questions 7 and 8**

7 A transformation  $A$  is represented by the matrix  $\mathbf{A}$  where  $\mathbf{A} = \begin{pmatrix} -1 & x & 2 \\ 7-x & -6 & 1 \\ 5 & -5x & 2x \end{pmatrix}$ .

The tetrahedron  $H$  has vertices at  $O, P, Q$  and  $R$ . The volume of  $H$  is 6 units.

$P', Q', R'$  and  $H'$  are the images of  $P, Q, R$  and  $H$  under  $A$ .

(a) In the case where  $x = 5$

- find the volume of  $H'$ ,
- determine whether  $A$  preserves the orientation of  $H$ . [3]

(b) Find the values of  $x$  for which  $O, P', Q'$  and  $R'$  are coplanar (i.e. the four points lie in the same plane). [4]

8 In this question you must show detailed reasoning.

$\mathbf{M}$  is the matrix  $\begin{pmatrix} 1 & 6 \\ 0 & 2 \end{pmatrix}$ .

Prove that  $\mathbf{M}^n = \begin{pmatrix} 1 & 3(2^{n+1} - 2) \\ 0 & 2^n \end{pmatrix}$ , for any positive integer  $n$ . [6]

**END OF QUESTION PAPER**

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