GCE

Further Mathematics A

Y535/01: Additional pure mathematics

AS Level

Mark Scheme for June 2024

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Question		Answer	Marks	AO	Guidance
1		$28A3B_{12} = 11 + (3\times12) + (10\times12^2) + (8\times12^3) + (2\times12^4)$	M1	1.1	**DR** Use of powers of 12 with A = 10 and B = 11. Condone one error in the expansion of powers of 12. Values of A and B can be implied by correct answer below.
		= 56 783	A1	1.1	
			[2]		

Question		Answer	Marks	AO	Guidance
2	(a)	Attempt at vector product of their **a** and **b**: $\begin{pmatrix}2\\4\\9\end{pmatrix}\times\begin{pmatrix}3\\-4\\6\end{pmatrix}$	M1*	1.1	Must be clear that it is the vector product being attempted. Can be implied by two correct components. If using Area $\Delta OAB = \frac{1}{2}\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$, attempt at $\lvert\mathbf{a}\rvert$, $\lvert\mathbf{b}\rvert$ and θ via the scalar product.
		$\mathbf{a}\times\mathbf{b} = \begin{pmatrix}60\\15\\-20\end{pmatrix}$	A1	1.1	Or **b × a** If using Area $\Delta OAB = \frac{1}{2}\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta\rvert$, $\lvert\mathbf{a}\rvert=\sqrt{101}$, $\lvert\mathbf{b}\rvert=\sqrt{61}$, $\theta = 55.9\ldots^0$
		$\lvert\mathbf{a}\times\mathbf{b}\rvert = \sqrt{60^2+15^2+(-20)^2}$	M1dep*	1.1	Attempt at the magnitude of their **a × b** soi Or Area $\Delta OAB = \frac{1}{2}\sqrt{101}\times\sqrt{61}\ \sin 55.9\ldots^0$
		Area $\Delta OAB = \frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert = \frac{65}{2}$ (square units)	A1	1.1	Triangle formula used **CAO**
			[4]		

Question		Answer	Marks	AO	Guidance
2	(b)	$\begin{pmatrix}2\\4\\3\lambda\end{pmatrix}\times\begin{pmatrix}\lambda\\-4\\6\end{pmatrix}=\begin{pmatrix}24+12\lambda\\3\lambda^2-12\\-8-4\lambda\end{pmatrix}=\mathbf{0}$	**M1**	**1.1**	Vector product attempted and result equated to **0.** Allow sign errors. Condone calculation of only one correct component equated to 0 leading to $\lambda = -2$.
		$\lambda = -2$	**A1**	**1.1**	A check that remaining two components are 0 is required. www
		ALT. Want **b** = m**a** (or **a** // **b**) and attempt at $m=\frac{\lambda}{2}=\frac{-4}{4}=\frac{6}{3\lambda}$	**M1**		
		$\lambda = -2$	**A1**		
			[2]		

Question			Answer	Marks	AO	Guidance
3	(a)		$f_x = 8xy - 6y^2 - \frac{1}{3}x^3$	**B1**	**1.1**	
			$f_y = 4x^2 - 12xy$	**B1**	**1.1**	
			$f_y = 0 \Rightarrow x = 3y$ (Noting $x \neq 0$ not required)	**M1**	**1.1**	Establishing a relationship between x and y by setting one first partial derivative to zero
			$f_x = 0 \Rightarrow 8.3y.y - 6y^2 - \frac{1}{3}.27y^3 = 0$	**M1**	**1.1**	Substituting correctly for first variable in second partial derivative set to zero **and** solving attempt of some cubic polynomial
			$\Rightarrow b = 2$	**A1**	**1.1**	Second variable value **CAO**
			$\Rightarrow a = 6$ and $c = 36$	**A1**	**1.1**	Both remaining values correct Condone x, y, z for a, b, c throughout

Question			Answer	Marks	AO	Guidance
				[6]		
	(b)	**i**	At U_1, $x = 5.9$, $y = 2$, $f_x = 1.94\ (= 8 \times 5.9 \times 2 - 6 \times 2^2 - \frac{1}{3} \times 5.9^3)$ At U_2, $x = 6.1$, $y = 2$, $f_x = -2.06\ (= 8 \times 6.1 \times 2 - 6 \times 2^2 - \frac{1}{3} \times 6.1^3)$	**B1** **B1** **[2]**	**1.1** **1.1**	If **B0B0**, **SCB1** for both correct substitutions
		ii	At V_1, $x = 6$, $y = 1.9$, $f_y = 7.2\ (= 4 \times 6^2 - 12 \times 6 \times 1.9)$ At V_2, $x = 6$, $y = 2.1$, $f_y = -7.2\ (= 4 \times 6^2 - 12 \times 6 \times 2.1)$	**B1** **B1** **[2]**	**1.1** **1.1**	If **B0B0**, **SCB1** for both correct substitutions
		iii	∩-shaped parabola on an *x*-*z* set of axes labelled or implied by an equation in (x,z) or $(x,\ f(x,2))$ ∩-shaped parabola on a *y*-*z* set of axes labelled or implied by an equation in (y,z) or $(y,\ f(6,y))$	**B1** **B1** **[2]**	**1.1** **1.1**	Condone positional vaguenesses Ditto

Question		Answer	Marks	AO	Guidance
4	(a)	$F_{n+1} = F_n + F_{n-1}$ $F_{n+2} = 2F_n + F_{n-1}$ $F_{n+3} = 3F_n + 2F_{n-1}$ or $F_{n+4} = F_{n+3} + F_{n+2}$ $= 2F_{n+2} + F_{n+1}$ $= 3F_{n+1} + 2F_n$	M1	2.1	Fibonacci recurrence relation used correctly at least twice
		$F_{n+4} = 5F_n + 3F_{n-1}$	A1 **[2]**	2.2a	**CAO** If **M0**, **SCB1** for trial and error with F_r and F_s checked for $r, s \geq 6$
	(b)	When n = 4, $F_4 = 3$ which is a multiple of 3 so result is true (or n = 8, $F_8 = 21$)	B1	1.1	Basic case
		Assume the result is true for $n = k$, where k is a multiple of 4 (or n=4k) Therefore $F_k = 3N$ (or $F_{4k} = 3N$) where N is a positive integer or F_k (or F_{4k}) is a multiple of 3 Using part **(a)**, $F_{k+4}(F_{4k+4})$ = (*their* 5) F_r + (*their* 3) F_s, r, s in terms of k, $4k - 4 < r, s < 4k + 4$, $r \neq s$ $F_{k+4} = 3(5N + F_{k-1})$ or $F_{4k+4} = 3(5N + F_{4k-1})$ which is also a multiple of 3	M1	2.1	Clearly reasoned inductive step (no need to note that all terms are integers)
		Since $F_4 = 3$ is a multiple of 3 and F_k a multiple of 3 when k is a multiple of 4 $\Rightarrow F_{k+4}$ a multiple of 3, it follows by induction that F_n is always a multiple of 3 if n is a multiple of 4	A1	2.4	Fully correct conclusion with consistent use of subscripts www

Question		Answer	Marks	AO	Guidance
			[3]		

Question		Answer	Marks	AO	Guidance
5	**(a)**	**I** = $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$	**B1** **[1]**	**1.2**	No need to observe that det(**I**) = 1 Accept **I**
	(b)	*G* consists of the powers of **P**, which has order 6	**B1**	**2.4**	oe such as $\mathbf{P}^6 = \mathbf{I}$. Can be implied by explicit calculation of $\boldsymbol{P}^6$. Or recognising that **P** is a clockwise rotation through 60°. Allow unsupported statement and without mention that no lower power of **P** gives **I**.
		(*n* =)6	**B1** **[2]**	**2.2a**	
	(c) i	$\mathbf{Q}^2 = \begin{pmatrix}-\frac{1}{2} & \frac{1}{2}\sqrt{3}\\ -\frac{1}{2}\sqrt{3} & -\frac{1}{2}\end{pmatrix}$ (= **RQR**)	**B1**	**1.1**	
		$\boldsymbol{QR} = \begin{pmatrix}-\frac{1}{2} & \frac{1}{2}\sqrt{3}\\ \frac{1}{2}\sqrt{3} & \frac{1}{2}\end{pmatrix}$ or $\boldsymbol{RQ} = \begin{pmatrix}-\frac{1}{2} & -\frac{1}{2}\sqrt{3}\\ -\frac{1}{2}\sqrt{3} & \frac{1}{2}\end{pmatrix}$	**B1**	**1.1**	
		$\boldsymbol{QR} = \boldsymbol{RQ}^2$ and $\boldsymbol{RQ} = \boldsymbol{Q}^2\boldsymbol{R}$	**B1**	**1.1**	
		$\mathbf{Q} = \begin{pmatrix}-\frac{1}{2} & -\frac{1}{2}\sqrt{3}\\ \frac{1}{2}\sqrt{3} & -\frac{1}{2}\end{pmatrix}$, $\mathbf{R} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$, **I** and *m* = 6	**B1**	**2.2a**	All six elements must be seen (explicitly as matrices or in terms of **Q** and **R)** and no others for this **B1**. (Condone failure to note that $\mathbf{Q}^3 = \mathbf{R}^2 = \mathbf{I}$.)

Question		Answer	Marks	AO	Guidance
			[4]		
	(c) ii	One of {**R**, **I**} (or {**R**, $\mathbf{R}^2$}), {**Q**, $\mathbf{Q}^2$, **I**} (or {**Q**, $\mathbf{Q}^2$,$\mathbf{Q}^3$}), {**QR**, **I**}, {**RQ**, **I**} Another correct subgroup. All correct and no others.	**B1** **B1** **B1** **[3]**	**2.2a** **1.2** **1.1**	One correct subgroup Ignore {**I**} and/or *H* if mentioned. Alternative forms may be used or matrices given instead. Condone absence of brackets.

5	**(d)**	**True**: Cyclic ⇒ Abelian, so *G* is Abelian or All powers of a single element commute ⇒ Abelian **True**: *G* is cyclic, generated by **P** **False**: *H* is not Abelian, since (e.g.) **QR** ≠ **RQ** **False**: Not Abelian ⇒ not cyclic, so *H* is not cyclic or There is no generator in *H*	**B1** **B1** **B1** **B1** **[4]**	**2.4** **2.5** **2.2a** **2.4**	Each answer must have a valid reason supporting it

Question			Answer	Marks	AO	Guidance
6	(a)	i	$n = 9k + 3$ and $n = 9k + 6$ or n=3 (mod 9) and n=6 (mod 9)	B1 [1]	1.2	Allow $n = 9k \pm 3$ Condone $9n + 3$ and $9n + 6$ (or $9n - 3$), not $n = 9k + r$
		ii	For $n = 9k + 3$, $f(n) = 1+2^{9k+3}+4^{9k+3} = 1+8.(2^9)^k+64.(2^9)^{2k}$	M1	3.1a	Writing 2^{r+3} as $2^r \times 2^3$ and 4^{r+3} as $4^r \times 4^3$
			$2^9 = 512 \equiv 1 \pmod{73}$	B1	1.1	Noted or used at any stage NB $262144=512^2$
			so that $f(n) \equiv 1 + 8.(1)^k + 64.(1)^{2k} \pmod{73}$	M1	3.1a	Considering f(n) mod 73 by substituting $(2^9)^k$ and $(2^9)^{2k}$ with 1
			$= 1 + 8 + 64 = 73 \equiv 0 \pmod{73}$ and f(n) is a multiple of 73	A1	3.2a	All correctly concluded
			Similarly, for $n = 9k + 6$, $f(n) = 1+64.(2^9)^k+4096.(2^9)^{2k}$	M1	1.1	Writing 2^{r+6} as $2^r \times 2^6$ and 4^{r+6} as $4^r \times 4^6$ (**or** suitably modified for $n = 9k - 3$)
			so that $f(n) \equiv 1+64.(1)^k+8.(1)^{2k} = 1+64+8 = 73$ $\equiv 0 \pmod{73}$ and f(n) is a multiple of 73	A1 [6]	3.2a	From fully correct working, including noting that $4096 \equiv 8 \pmod{73}$ NB $n = 9k + 6$ could be considered first.
	(b)		If $n = 9k$, then $f(n) = 1+512^k+(512^2)^k$ or $1 + 512^k + (262144)^k$	M1	1.1	Correct set up using indices
			$\equiv 1 + 1^k + (1^2)^k = 3 \pmod{73}$	A1 [2]	2.2a	**CAO** It must be clear that this is always the case If **M0**, **SC1** substituting n=9 and evaluating mod 73 to find 3: $f(9)=1+2^9+4^9$ (=262657) $\equiv 3 \pmod{73}$

Question		Answer	Marks	AO	Guidance
7	(a)	If $E_0 = H_0 = 1200$ and $H_{n+1} = \left(1 - \frac{1}{8}\right) H_n$, then $H_{n+6} = \left(\frac{7}{8}\right)^6 H_n$	B1	3.3	H_n being the hourly amount of enzyme Adequately explained in words and/or symbols.
		and $E_{n+1} = \left(\frac{7}{8}\right)^6 E_n$ plus a 'boost' of 500	B1 [1]	2.4	
	(b)	Gen. Soln. is given by (E_n =) CS (+) PS	M1	1.1	Including good attempts at CS = $a \times r^n$ (with 1 arbitrary constant) and PS = constant
		CS = $a\left[\left(\frac{7}{8}\right)^6\right]^n$	A1	1.1	
		PS = $\dfrac{500}{1-\left(\frac{7}{8}\right)^6}$	A1	1.1	
		Use of $E_0 = 1200$ to evaluate their "a"	M1	3.4	Must be E_n = CS + PS PS must be numerical
		$E_n = \left(1200 - \dfrac{500}{1-\left(\frac{7}{8}\right)^6}\right) \times \left(\frac{7}{8}\right)^{6n} + \dfrac{500}{1-\left(\frac{7}{8}\right)^6}$	A1 [5]	1.1	Allow approximate numerical equivalents; e.g. $292.9 \times 0.4488^n + 907.1$
	(c)	$\left(\frac{7}{8}\right)^n$ or $\left(\frac{7}{8}\right)^{6n} \to 0$ as $n \to \infty$	B1	2.4	Condone explanation that $\left(\frac{7}{8}\right)^n$ or $\left(\frac{7}{8}\right)^{6n}$ becomes small (negligible) as $n \to \infty$.
		$\Rightarrow E_n \to 907$	B1 [2]	2.2b	Correct to **3sf**

Question		Answer	Marks	AO	Guidance
7	(d)	From $H_0 = 1200$, the hourly sequence H_n runs 1050, 918.75, 803.91, 703.42, … Attaining $H_6 = 538.55 \rightarrow E_1 =$ **1038(.55)**	M1	3.1a	**DR** Breaking the problem down into single 1-hour steps, multiplying by 0.875 each time. Making the jump from the end of the first six-hour period to the start of the second. Sight of 1038(.55) or $1200 \times \left(\frac{7}{8}\right)^6 + 500$ **FT** their solution from part **(b)**
		Continuing one hour at a time; so $H_7 = 908.7$, $H_8 = 795.14$, $H_9 = 695.75$, $H_{10} = 608.78$, $H_{11} = 532.68$, $H_{12} =$ **466.10**	A1	3.5a	Continue as before to H_{12}. Sight of 466(.10) with (numerical) justification. **FT** their solution from part **(b)** if their $H_{12}<500$
		Explaining that, just before the second 500 (mg) is added, the amount of enzyme **has fallen below** the required minimum amount, so the experiment's validity has failed	A1 **[3]**	3.5b	'466<500 so requirement not met'

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