Oxford Cambridge and RSA

GCE

Further Mathematics A

Y531/01: Pure Core

AS Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. ‘can’t do’, ‘don’t know’)
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn’t an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

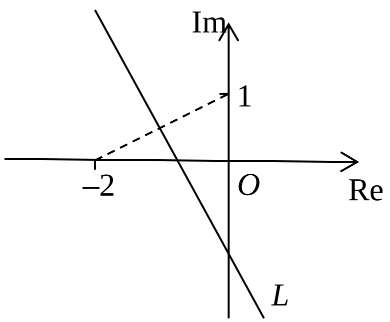
j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			$\begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & -4 \\ 5 & 6 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 40 \\ 61 \end{pmatrix}$	**M1**	1.1	Reduction of system to matrix form soi	
			$\begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & -4 \\ 5 & 6 & -1 \end{pmatrix}^{-1} = \frac{1}{125}\begin{pmatrix} 26 & 3 & 14 \\ -19 & -7 & 9 \\ 16 & -27 & -1 \end{pmatrix}$	**B1***	1.1	BC. Could be embedded Inverse matrix correctly evaluated	$\begin{pmatrix} 0.208 & 0.024 & 0.112 \\ -0.152 & -0.056 & 0.072 \\ 0.128 & -0.216 & -0.008 \end{pmatrix}$
			$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{125}\begin{pmatrix} 26 & 3 & 14 \\ -19 & -7 & 9 \\ 16 & -27 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 40 \\ 61 \end{pmatrix}$	**M1**	1.1	Correctly using the inverse matrix: (**r** =) $\mathbf{A}^{-1}\mathbf{b}$. It must be clear that a matrix method is being used.	Can be incorrect inverse for M1 Allow M1 for expressions of the form $\mathbf{A}^{-1}\mathbf{b}$
			$x = 8$, $y = 2$, $z = -9$	**A1*dep** **[4]**	1.1	Could be seen in vector form but x, y and z must be appropriately seen.	Correct answer with no matrix forms shown (with or without other working) is 0/4. Need to have earned the B1

Question			Answer	Marks	AO	Guidance	
2	(a)		**DR** $\frac{8+i}{2-i}\times\frac{2+i}{2+i}=\frac{16+8i+2i-1}{4+1}$	**M1**	1.1	Multiplying top and bottom by the conjugate of the bottom and attempting expansion involving $i^2 = -1$.	Allow appearance of 5 in denominator with no working shown as long as clear what numerator multiplied by.
			$=\frac{15+10i}{5}=3+2i$ cao	**A1** **[2]**	1.1	Answer must be in the requested form.	Must have at least one line of working before answer (DR)
	(a)		**DR Alternate method** $(8+i)=(2-i)(a+bi)$ $8=2a+b$ $1=-a+2b$	**M1**		Equating to a+bi and rearranging to real and imaginary parts	
			$\Rightarrow a=3, b=2$ 3+2i cao	**A1**		Answer must be in the requested form.	Must have at least one line of working before answer (DR)
	(b)		**DR** $x=\frac{-(-8)\pm\sqrt{(-8)^2-4\times4\times5}}{2\times4}=\frac{8\pm\sqrt{-16}}{8}=\frac{8\pm4i}{8}$	**M1**	1.1	Use of formula and finding the square root of a negative number in terms of i (can be awarded even if error in calculation under square root).	Condone one sign error (eg –8 rather than –(–8) or $(-8)^2 = -64$). Could also use completing the square. Must get as far as attempting "x=…" but might make some slips.
			$=1+\frac{1}{2}i$ or $1-\frac{1}{2}i$	**A1** **[2]**	1.1	or $1\pm\frac{1}{2}i$ or $1 \pm 0.5i$ but must be in correct form so eg $\frac{2\pm i}{2}$ is **A0**	Condone $1\pm\frac{i}{2}$

Question			Answer	Marks	AO	Guidance	
3	(a)	(i)	$\begin{pmatrix}1\\2\\-1\end{pmatrix}\times\begin{pmatrix}3\\5\\-2\end{pmatrix}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	B1 [1]	1.1		
		(ii)	$\begin{pmatrix}1\\-1\\-1\end{pmatrix}$ is perpendicular to [both] $\begin{pmatrix}1\\2\\-1\end{pmatrix}$ and [also] $\begin{pmatrix}3\\5\\-2\end{pmatrix}$	B1FT [1]	1.2	This can be stated as a generality (eg **a×b** is perpendicular to both **a** and also **b**). FT their answer to **(a)(i)** (ie the mark can be awarded if the property is stated for their vector which is not, in fact, perpendicular).	Accept "perpendicular to both vectors" or even just "they are perpendicular" But if vectors stated must be correct vectors (or their vectors if MR in part **(i)**). If cross product vector stated must be their cross product vector.
		(iii)	If the dot product is zero then the vectors are perpendicular	M1	2.1	**M1** can be implied by Attempt to find a relevant dot product and show that this is 0	
			$\begin{pmatrix}1\\2\\-1\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=1-2+1=0$ $\begin{pmatrix}3\\5\\-2\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=3-5+2=0$ (so the answer to **(a)(i) is** perpendicular to both as claimed.)	A1 [2]	2.2a	Correct calculation of **a.**(**a×b**) and **b.**(**a×b**), showing some details of calculation (ie not simply stating "= 0" without justification).	NB Both these marks are still available to candidates whose answer to **(a)(i)** is $\begin{pmatrix}-1\\1\\1\end{pmatrix}$.

Question			Answer	Marks	AO	Guidance	
	(b)		$(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}).(4\mathbf{i} - \mathbf{j} + 8\mathbf{k}) = 8 + 2 + 8 = 18$	B1	1.1	Can be implied by correct answer	
			$\cos\theta = \frac{a.b}{\lvert a\rvert\lvert b\rvert}$ $= \frac{18}{\sqrt{2^2+(-2)^2+1^2}\sqrt{4^2+(-1)^2+(-8)^2}}$	M1	1.1	Correct method for evaluation of cosine of required angle, including correct form for both moduli.	**M1** can be awarded after correct rearrangement of $\mathbf{a.b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ once correct form for moduli seen even if subsequent (calculation) error.
			$= \frac{18}{\sqrt{9}\sqrt{81}} = \frac{2}{3}$ $\therefore \theta = cos^{-1}\left(\frac{2}{3}\right) = 48.2° \ (1\ dp)$	A1	1.1	awrt 48.2° or 0.841 rads.	48.1896851… or 0.8410686706
				[3]			

Question			Answer	Marks	AO	Guidance	
4	(a)		$\lvert z-(4-2\text{i})\rvert$	**M1**	1.1	Any solution which involves $\lvert z-(4-2\text{i})\rvert$ or $\lvert z-4+2\text{i}\rvert$	M1 for $(x-4)^2 + (y+2)^2 = 9$
			$\{z : z \in \mathbb{C},\ \lvert z-(4-2\text{i})\rvert = 3\}$	**A1** **[2]**	2.5	Must show the {} brackets and correct set notation used, i.e. the form shown but BOD lack of comma.	or $\left\{\begin{matrix} z = x+y\text{i} : x, y \in \mathbb{R}, \\ (x-4)^2+(y+2)^2 = 9 \text{ (or } 3^2) \end{matrix}\right\}$
	(b)		(need points equidistant from) i and –2	**B1**	1.1	i and –2 both clearly identified either on the sketch or in words.	Ignore other points.
				**B1FT** **[2]**	1.1	Their two identified points (joined and) perpendicularly bisected by a single, straight, solid line which should be labelled *L* or otherwise unambiguously indicated. If points not explicitly identified then 1st **B1** can be implied by either correct equation of line or both intercepts of line given.	NB The equation of this line (which need not be shown) is $y = -2x - 3/2$ so the x- & y-intercepts are –¾ and –1½ respectively. These need not be shown but if shown must be correct. Line must be indicated as perpendicular or implied to be per perpendicular e.g. from two correct points on line or correct equation. **SC** If B0 for not labelling the points but the points are located correctly, and line indicated appears to be the perpendicular bisector then allow B1

	(c)		Either $x = 3$ or $\tan^{-1}\left(\frac{y}{x-1}\right) = \frac{1}{4}\pi$ stated or indicated.	**M1**	3.1a	Understanding of one of the conditions. Could be shown on a diagram (eg the line $x = 3$ drawn or 3 as a clear, special label on the real axis or the half-line with gradient 1 drawn from (1, 0), etc)	$y = x$ -1 implies M1 answer of 3 + *b*i implies M1
			$\tan^{-1}\left(\frac{y}{x-1}\right) = \frac{1}{4}\pi \Rightarrow \frac{y}{3-1} = 1 \Rightarrow y = 2$ so the number is 3 + 2i	**A1** **[2]**	3.2a	Could be shown on a diagram	Find - might not be much (or even any) working shown

Question			Answer	Marks	AO	Guidance	
5	**(a)**		$\overrightarrow{AB} = \begin{pmatrix} 11 \\ -9 \\ 0 \end{pmatrix} - \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$	**M1**	1.1	Subtracting the position vectors of *A* and *B* (in either order). Calculation or answer is enough for M1	Allow M1 for a row vector i.e. (3, -2, 2)
			$\mathbf{r} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$	**A1** **[2]**	1.1	Must be "$\mathbf{r} =$" (or "$\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$") Allow r or $\mathbf{L_1}$ but not L_1 (i.e. allow r not to be underlined as a vector, but if L_1 used this must be indicated as a vector.	$\mathbf{r} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix}$ or $\begin{pmatrix} 11 \\ -9 \\ 0 \end{pmatrix} \pm \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ Note – could use other "starting points" Must be column vectors here

Question			Answer	Marks	AO	Guidance	
	(b)		$8 + 3\lambda = 26$ or $-7 - 2\lambda = -19$ or $-2 + 2\lambda = -14$	M1	1.1	Writing down a correct equation for any component of their declared line.	Note – values of λ will depend on the "starting point" used in part **(a)** and different multiples of the direction vector.
			(x gives $\lambda = 6$ or y gives or $\lambda = 6$) [but] z gives $\lambda = -6$ so (26, –19, –14) does **not** lie on the line	A1FT **[2]**	2.2a	Finding a correct inconsistency and reaching correct conclusion. Not all values need be given but values given must be correct. FT their declared line.	Could see eg $\lambda = 6 \Rightarrow$ ($y = -19$ but) $z = 10$ which is not –14 Do not need to see the word "inconsistent" or explicit comparison. Finding two different values of λ and then stating does not lie on line is enough
	(b)		**Alternate method** $\begin{pmatrix} 18 \\ -12 \\ -12 \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$	M1		Rearranging vector equation	
			But $\begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ is not parallel to $\begin{pmatrix} 18 \\ -12 \\ -12 \end{pmatrix}$ so the point does not lie on the line	A1FT		Or finding λ inconsistency as before	A0 for "there is no value of λ that works" without justification

	(c)		$\overrightarrow{OC}=\begin{pmatrix}8\\-7\\-2\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\2\end{pmatrix}$ [for some particular value of λ.]	B1FT	3.1a	Can be embedded or implied. Can be same or different symbol as parameter in **(a)**.	
			$[\overrightarrow{OC}=\mu\begin{pmatrix}a\\b\\c\end{pmatrix}]$ and $\begin{pmatrix}a\\b\\c\end{pmatrix}.\begin{pmatrix}3\\-2\\2\end{pmatrix}=0$	B1FT	2.1	Condition for perpendicularity stated.	Don't have to see an equation for OC in terms of μ here.
			$3a-2b+2c=0$ $3(8+3\lambda)-2(-7-2\lambda)+2(-2+2\lambda)=0$	M1	1.1	Forming a correct dot product and using it with the other definition of $\overrightarrow{OC}$ to form an equation in the parameter.	μ may or may not be present here, but must be dealt with appropriately for the A mark.
			$\Rightarrow \lambda=-2 \Rightarrow \overrightarrow{OC}=\begin{pmatrix}8\\-7\\-2\end{pmatrix}-2\begin{pmatrix}3\\-2\\2\end{pmatrix}=\begin{pmatrix}2\\-3\\-6\end{pmatrix}$ so the equation of L_2 is $\mathbf{r}=\mu\begin{pmatrix}2\\-3\\-6\end{pmatrix}$	A1 **[4]**	2.2a	Condone use of the same parameter as L_1. Could see eg $\mathbf{r}=\begin{pmatrix}2\\-3\\-6\end{pmatrix}+\mu\begin{pmatrix}2\\-3\\-6\end{pmatrix}$	BOD lack of "$\mathbf{r}=$" (or "$\begin{pmatrix}x\\y\\z\end{pmatrix}=$") in this part if has already been penalised in part (a). If full marks awarded in part (a) then must have correct notation here for full marks.

	(c)		**Alternative Method:**				
			$\overrightarrow{OC} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ for some particular value of λ.	**B1FT**		Can be embedded or implied. Can be same or different symbol as parameter in **(a)**.	
			$\overrightarrow{OC}.\mathbf{b}_{L_1} = 0$	**B1FT**		Condition for perpendicularity stated.	
			$\therefore \left(\begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} \right) . \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ $= 24 + 14 - 4 + (9 + 4 + 4)\lambda = 0$	**M1**		Forming a correct dot product and using it to form an equation in the parameter.	$\therefore \begin{pmatrix} 8+3\lambda \\ -7-2\lambda \\ 10+2\lambda \end{pmatrix} . \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ $= 24 + 9\lambda + 14 + 4\lambda + 20 + 4\lambda = 0$
			$\Rightarrow \lambda = -2 \Rightarrow \overrightarrow{OC} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} - 2 \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$ so the equation of L_2 is $\mathbf{r} = \mu \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$	**A1**		Could see eg $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ -6 \end{pmatrix}$ Condone use of the same parameter as L_1.	Condone presence of zero vector as first point BOD lack of "**r** =" (or "$\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$") in this part if has already been penalised in part (a). If full marks awarded in part (a) then must have correct notation here for full marks
	(c)		**Alternative Method 2:**				
			$\overrightarrow{OC} = \begin{pmatrix} 8 \\ -7 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ for some particular value of λ.	**B1FT**		Can be embedded or implied. Can be same or different symbol as parameter in **(a)**.	

			Distance $\lvert\overrightarrow{OC}\rvert^2$ is given by: $\lvert\overrightarrow{OC}\rvert^2$ $= (8+3\lambda)^2 + (-7-2\lambda)^2 + (-2+2\lambda)^2$ $= 64 + 48\lambda + 9\lambda^2 + 49 + 28\lambda + 4\lambda^2 + 4 - 8\lambda + 4\lambda^2$ $= 17\lambda^2 + 68\lambda + 117$ Equating the derivative to 0 gives $34\lambda + 68 = 0 \Rightarrow \lambda = -2$ => $\lambda = -2$ => $\overrightarrow{OC} = \begin{pmatrix}8\\-7\\-2\end{pmatrix} - 2\begin{pmatrix}3\\-2\\2\end{pmatrix} = \begin{pmatrix}2\\-3\\-6\end{pmatrix}$ so the equation of L_2 is $\mathbf{r} = \mu\begin{pmatrix}2\\-3\\-6\end{pmatrix}$	M1 M1 A1		Using the fact that minimum distance is the perpendicular distance. Condone use of the same parameter as L_1. Condone presence of zero vector as first point Could see eg $\mathbf{r} = \begin{pmatrix}2\\-3\\-6\end{pmatrix} + \mu\begin{pmatrix}2\\-3\\-6\end{pmatrix}$	BOD lack of "**r** =" (or "$\begin{pmatrix}x\\y\\z\end{pmatrix}$ =") in this part if has already been penalised in part (a). If full marks awarded in part (a) then must have correct notation here for full marks
	(d)		Because C lies on L_1 and L_2 and OC is perpendicular to L_1, OC must be the shortest route from O to L_1. $\lvert\overrightarrow{OC}\rvert = \sqrt{2^2 + (-3)^2 + (-6)^2} = \sqrt{49} = 7$ so shortest distance from O to L_1 is 7 units.	M1 A1 **[2]**	3.1a 1.1	Can be implied by sight of $\lvert\overrightarrow{OC}\rvert$. Or attempt at evaluating |OC| Follow through sign errors on coordinates of C	If just an evaluation seen need to see the vector identified as C in previous part (could be from equation). i.e. do not allow method mark for modulus of a vector which is not C.

Question			Answer	Marks	AO	Guidance	
6			Basis case: $n = 1$ $\mathbf{A}^1 = \begin{pmatrix} 1 & a \times 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ [$=\mathbf{A}$] [So it's true when n=1]	**B1**	2.1	$\mathbf{A}^1$ and $a\times1$ must both be seen explicitly.	Accept "=**A**" instead of "therefore true when n=1"
			Assume true for $n = k$ so assume $\mathbf{A}^k = \begin{pmatrix} 1 & ak \\ 0 & 1 \end{pmatrix}$	**M1**	2.1	Setting up the inductive hypothesis properly.	
			$\therefore \mathbf{A}^{k+1} = \mathbf{A}^k\mathbf{A} = \begin{pmatrix} 1 & ak \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ by our inductive hypothesis.	**M1**	1.1	Considering $\mathbf{A}^{k+1}$ and using their inductive hypothesis. Could also consider $\mathbf{A}\mathbf{A}^k$	Jumping to $\mathbf{A}^{k+1} = \begin{pmatrix} 1 & a(k+1) \\ 0 & 1 \end{pmatrix}$ without seeing the two matrices multiplied together first scores **M0**.
			$\therefore \mathbf{A}^{k+1} = \begin{pmatrix} 1 & a+ak \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a(k+1) \\ 0 & 1 \end{pmatrix}$	**A1**	2.2a	Must see a+ak appear before being factorised.	
			So true for $n = k$ => true for $n = k + 1$. But true for $n = 1$. Therefore true for all [integer] $n \geq 1$.	**A1** **[5]**	2.4	Clear conclusion for induction process. Must mention both basis case and that statement true for k implies true for $k + 1$. BOD missing the word integer	If B0 not awarded as $\mathbf{A}^1$ or $a\times1$(or both) not seen allow A1 here (but must have attempt at base case)

Question			Answer	Marks	AO	Guidance	
7			**DR** $\alpha\beta\gamma=-\frac{5}{2}, \sum\alpha\beta=-\frac{3}{2}, \sum\alpha=\frac{3}{2}$	**M1**	1.1	Attempt to find the three Vieta's formulae for the original equation.	This mark can be awarded if 2 of the 3 are correct and the 3rd has been attempted or if all are correct but for signs.
			$\alpha'\beta'\gamma'=\frac{\alpha\beta}{\gamma}\times\frac{\beta\gamma}{\alpha}\times\frac{\gamma\alpha}{\beta}=\alpha\beta\gamma\left(=-\frac{5}{2}\right)$	**B1**	1.1	Showing that the new product of roots is the same as the original.	
			$(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$ $(\alpha\beta+\beta\gamma+\gamma\alpha)^2=$ $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2+2(\alpha\beta^2\gamma+\beta\gamma^2\alpha+\gamma\alpha^2\beta)$ $=\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2+2\alpha\beta\gamma(\alpha+\beta+\gamma)$	**M1**	1.1	Both considered and at least one correctly written in desired form using Vieta's expressions	May be embedded in expressions for $\sum\alpha'\beta'$ or $\sum\alpha'$
			$\sum\alpha'\beta'=\frac{\alpha\beta}{\gamma}\frac{\beta\gamma}{\alpha}+\frac{\beta\gamma}{\alpha}\frac{\gamma\alpha}{\beta}+\frac{\gamma\alpha}{\beta}\frac{\alpha\beta}{\gamma}$ $=\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ $=\left(\frac{3}{2}\right)^2-2\times\left(-\frac{3}{2}\right)=\frac{9}{4}+\frac{12}{4}=\frac{21}{4}$	**B1**	1.1	For correct $\sum\alpha'\beta'$ in terms of Vieta's expressions i.e. $\sum\alpha'\beta'=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$	Can be awarded before numerical value found Correct numerical value implies correct $\sum\alpha'\beta'$ (but must be identified as $\sum\alpha'\beta'$)

			$\sum \alpha' = \frac{\alpha\beta}{\gamma} + \frac{\beta\gamma}{\alpha} + \frac{\gamma\alpha}{\beta} = \frac{\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2}{\alpha\beta\gamma}$ $= \frac{(\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)}{\alpha\beta\gamma}$ $= \frac{(\alpha\beta + \beta\gamma + \gamma\alpha)^2}{\alpha\beta\gamma} - 2(\alpha + \beta + \gamma)$ $= \frac{\left(-\frac{3}{2}\right)^2}{\left(-\frac{5}{2}\right)} - 2\times\left(\frac{3}{2}\right) = -\frac{9}{4}\times\frac{2}{5} - \frac{30}{10} = -\frac{39}{10}$	**B1**	1.1	For correct $\sum \alpha'$ in terms of Vieta's expressions i.e. $\sum\alpha' = \frac{(\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)}{\alpha\beta\gamma}$	Can be awarded before numerical value found Correct numerical value implies correct $\sum \alpha'$ (but must be identified as $\sum \alpha'$)
			Choosing $a' = 20$ gives $20x^3 + 78x^2 + 105x + 50 = 0$	**A1** **[6]**	1.1	Any non-zero integer multiple. Must be a cubic ***equation*** (i.e. it must have "=0") with integer coefficients but can be in any unknown.	Correct answer from finding roots on calculator only scores 0/6.

Question			Answer	Marks	AO	Guidance	
8	(a)		$\mathbf{A}^{-1} = \frac{1}{3}\begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$	B1 [1]	1.1		
	(b)		$[\mathbf{AB} =]\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & -3 \end{pmatrix}$ $[\mathbf{BA} =]\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & -3 \end{pmatrix}$ so they are not the same	M1 A1 [2]	1.1 2.2a	Correctly finding either **AB** or **BA**. Must give correct calculation and correct conclusion.	$\mathbf{AB} \neq \mathbf{BA}$ is fine for conclusion. General statement "matrices not commutable not OK unless linked to particular case.
	(c)		The matrix representing R is **C**(**BA**)...	M1	3.1a	Or the matrix representing S is (**CB**)**A** Must have **R=C(BA)** or **S=(CB)A** with brackets or equivalent correct	Need to see the brackets or equivalent BOD sight of T_A
			...and the matrix representing S is (**CB**)**A** and by associativity (of matrix multiplication), **C**(**BA**) = (**CB**)**A** (so R and S are the same)	A1 [2]	3.2a	Correct form for S and either explicit equality or associativity property explicitly mentioned. M1 A0 only if state that matrices are commutative	If **M0** then **SC B1** for observation that (**AB**)**C** = **A**(**BC**) or any other correct statement of associativity of matrix multiplication.

Question			Answer	Marks	AO	Guidance	
	(d)	(i)	det**A** = 3 or det**B** = –1	**M1**	1.1	Correctly calculating the determinant of A or B	Question says "Determine" so answer only is 0
			det (**CBA**) = $-3(a^2 + 1) < 0$ (since a is real) [so the orientation is reversed]. Order is *D', G', F', E'*	**A1** **[2]**	2.2a	Some justification must be given but condone incorrect order of matrices. Orientation reversed is fine, but must see correct det (**CBA)** **SC1** If neither det **A** or det **B** explicitly calculated then allow B1 for det **A**, det **C** positive and det **B** negative so orientation reversed.	Can consider det(**C**) det(**B**) det(**A**) instead, i.e. the effect on orientation of each of the three transformations Allow "the order is clockwise". **SC2** If only det (C) considered (i.e. candidate thinks transformation R is represented by **C**) then allow **B1** for $a^2+1>0$ so orientation is the same
		(ii)	$15(a^2 + 1)$	**B1FT** **[1]**	1.1	FT 5× | their determinant from **(i) | (**if found)	FT only if their determinant is negative
		(iii)	The determinant is not zero...	**M1**	1.1	Understanding that the matrix associated with the transformation is non-singular. Allow M1 for "the determinant is negative". Or equating det to 0 and solving even if conclusion wrong	Can be considering det **C** or det **R** here Allow M1 for "determinant is positive" ONLY if it is clear where the determinant has come from (e.g. from det **C**).

			...so the inverse transformation exists.	**A1FT** **[2]**	2.2a	Condone use of "matrix" rather than "transformation" (and vice versa as applicable). Allow follow through if they are using det **C.** If have found matrix singular when a = i then need to discount this as not real.	If their determinant is 0 then **SC1** only can be awarded for showing understanding that the transformation associated with a singular matrix does not have an inverse. Need the be showing non-zero determinant.

Question			Answer	Marks	AO	Guidance	
9			**DR** $a = 0$ (is one possibility)	**B1**	1.1		
			$1^4 + 1^3 + 3\times1^2 - 5\times1 = 0$ so $a = 1$ (is another possibility)	**B1**	3.1a	or eg f(1) = 0 if intent is clear but must be some justification.	
			$x^4 + x^3 + 3x^2 - 5x = x(x^2(x-1) + 2x(x-1) + 5(x-1)) = x(x-1)(x^2 + 2x + 5)$ and discriminant of quadratic $= 2^2 - 4\times1\times5 = -16 < 0$ so no further real roots.	**B1**	2.3	Some justification must be given that there are no more real roots.	Allow for finding the two complex roots ($-1 \pm 2i$) - must have seen the correct $x^2 + 2x + 5$ If **B1B0B0** or **B0B0B0** then **SC1** for "a = 1 and no others" or "a=1, ($-1 \pm 2i$)" without justification.

			$a = 0 \Rightarrow 2 + 3i$ is a root of $z^4 - 2z^3 - 10z^2 + 86z - 195$ [= 0] so $2 - 3i$ is also a root OR $a = 1 \Rightarrow 3 + 3i$ is a root of $z^4 - 4z^3 + 11z^2 + 6z + 90$ [= 0] so $3 - 3i$ is also a root	**M1**	3.1a	Condone small errors in calculation of coefficients in equation	Need both the pair of complex roots SOI and the quartic shown (allow sign slips). Only need one case for the M1.
			$2 + 3i + 2 - 3i = 4$ and $(2 + 3i)(2 - 3i) = 13$ so $z^2 - 4z + 13$ is a factor OR $3 + 3i + 3 - 3i = 6$ and $(3 + 3i)(3 - 3i) = 18$ so $z^2 - 6z + 18$ is a factor	**M1**	1.1	oe eg expanding $(z - (2 + 3i))(z - (2 - 3i))$ or $(z - (3 + 3i))(z - (3 - 3i))$	Attempt to find quadratic factor from the complex roots. Only one case needed for M1 Allow with no working
			$z^4 - 2z^3 - 10z^2 + 86z - 195 = (z^2 - 4z + 13)(z^2 + 2z - 15)$ OR $z^4 - 4z^3 + 11z^2 + 6z + 90 = (z^2 - 6z + 18)(z^2 + 2z + 5)$	**M1**	1.1	Attempt to factorise their quartic with their quadratic factor (at least z^3 and constant terms consistent).	Only one case needed. MUST see some evidence of factorisation here
			$z^2 + 2z - 15 = 0 \Rightarrow z = -5$, $z = 3$ and $2 \pm 3i$ stated as roots (possibly earlier).	**A1**	2.2a	All four roots SOI for the a=0 case	**DR** so need to see evidence of where the roots came from i.e. factorisation into two quadratics If extra values of z found (from complex a or incorrect a values) then A0
			$z^2 + 2z + 5 = 0 \Rightarrow z = -1 \pm 2i$ and $3 \pm 3i$ stated as roots (possibly earlier).	**A1**	2.2a	All four roots SOI for the a=1 case	
				[8]			

			Alternate for 1st and 2nd M mark				
			$(a+2+3i)+(a+2-3i)=2a+4$ and $(a+2+3i)(a+2-3i)=(a+2)^2+9$ so $z^2-(2a+4)z+[a^2+4a+13]$ is a factor	**M1**		Quadratic factor found in general case. Can also be found by expanding $[z-(a+2+3i)][z-(a+2-3i)]$	Constant term can be either $(a+2)^2+9$ or $[a^2+4a+13]$
			$a=0 \Rightarrow z^2-4z+13$ is a factor OR $a=1 \Rightarrow z^2-6z+18$ is a factor	**M1**			
			Alternate 2 if only the quartic in *z* with *a*=0 considered via linear factors				
			$a=0$ (is one possibility)	**B1**			
			$1^4+1^3+3\times1^2-5\times1=0$ so $a=1$ (is another possibility)	**B1**		or eg f(1) = 0 if intent is clear but must be some justification.	
			$x^4+x^3+3x^2-5x=x(x^2(x-1)+2x(x-1)+5(x-1))=x(x-1)(x^2+2x+5)$ and discriminant of quadratic $=2^2-4\times1\times5=-16<0$ so no further real roots.	**B1**		Some justification must be given that there are no more real roots. Allow for finding the two complex roots $(-1\pm2i)$ (must have seen correct quadratic)	If **B1B0B0** or **B0B0B0** then **SC1** for "$a=1$ and no others" or "$a=1$, $(-1\pm2i)$" without justification.
			If $a=0$ quartic is $z^4-2z^3-10z^2+86z-195$ f(3)=0 so $(z-3)$ is a factor $f(z)=(z-3)(z^3+z^2-7z+65)$	**M1**		Identifying linear factor and factorising	
			f(-5)=0 so $(z+5)$ is a factor $f(z)=(z-3)(z+5)(z^2-4z+13)$	**M1**			
			So the roots are 3, -5, 2+3i, 2-3i	**A1**		Identifying 4 roots	Final 2 marks unavailable

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