Oxford Cambridge and RSA

GCE

Further Mathematics A

Y531/01: Pure Core

Advanced Subsidiary GCE

Mark Scheme for November 2020

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. ‘can’t do’, ‘don’t know’)
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn’t an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			**DR** $(a + b\text{i})^2 = a^2 - b^2 + 2ab\text{i}$	**B1**	1.1	Seen or implied in solution	
			$a^2 - b^2 = -77$ and $2ab = -36$ (where a and b are real)	**M1**	1.1	Comparing real and imaginary parts (no i unless later recovered) from a 3 term expansion	Allow equating real and imaginary considering $(a+\text{i}b)(c+\text{i}d)$
			$b = -\frac{18}{a} \Rightarrow a^2 - \left(-\frac{18}{a}\right)^2 = -77$ $\Rightarrow a^4 + 77a^2 - 324 = 0$	**M1**	1.1	Eliminating b or a to obtain 3 term quadratic in a^2 or b^2. Unknowns must not be in denominator and non-zero terms on same side. = 0 seen or implied by solution.	$(b^4 - 77b^2 - 324 = 0)$ Factorised forms: $(b^2 - 81)(b^2 + 4)$
			$(a^2 - 4)(a^2 + 81) = 0$	**B1**	2.1		DR requires evidence of solving quadratic in a^2 (or b^2). Can be implied by sight of all 4 solutions.
			$a^2 = 4$ or $b^2 = 81$ only	**A1ft**	2.3	Rogue solutions; $a^2 = -81$, $b^2 = -4$. Any rogue solutions must be discarded before A1 awarded.	For follow through need to discard rogue solutions
			$2 - 9\text{i}$ and $-2 + 9\text{i}$	**A1** **[6]**	1.1	Both roots. Can use ± but not $\pm 2 \pm 9\text{i}$ and not $\pm 2 - 9\text{i}$. $\pm(2 - 9\text{i})$, $\pm(-2 + 9\text{i})$ or $\pm 2 \mp 9\text{i}$ are all acceptable.	$2 - 9\text{i}$ and $-2 + 9\text{i}$ without working from quartic could score B1M1M1B0A0A1

Question			Answer	Marks	AO	Guidance	
2	(a)		$\mathbf{A}=\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	B1 [1]	1.2		
	(b)		$\begin{pmatrix}. & 0\\. & 1\end{pmatrix}$ $\begin{pmatrix}1 & .\\2 & .\end{pmatrix}$	B1 B1 [2]	1.1 1.1	$\begin{pmatrix}0\\1\end{pmatrix}\to\begin{pmatrix}0\\1\end{pmatrix}$ ie y-axis invariant $\begin{pmatrix}1\\0\end{pmatrix}\to\begin{pmatrix}1\\2\end{pmatrix}$	$\begin{pmatrix}1 & 0\\. & .\end{pmatrix}$ or $\begin{pmatrix}. & .\\2 & 1\end{pmatrix}$ insufficient for B1
	(c)		$\mathbf{C}=\begin{pmatrix}1 & 0\\2 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ $=\begin{pmatrix}1 & 0\\2 & -1\end{pmatrix}$	M1 A1 [2]	1.1 1.1	Their **A** and **B** but must be the correct way round (ie **BA**, not **AB**).	
	(d)		$\begin{pmatrix}1 & 0\\2 & -1\end{pmatrix}\begin{pmatrix}x\\x\end{pmatrix}=\begin{pmatrix}x\\x\end{pmatrix}$ Since each point gets mapped to itself it is a line of invariant points	M1 A1 [2]	3.1a 2.2a	Multiplying a correct vector ($\begin{pmatrix}x\\x\end{pmatrix}$ or $\begin{pmatrix}y\\y\end{pmatrix}$) correctly into C. If M0 then SC1 for use of a particular point leading to correct conclusion (This can follow from an incorrect matrix, possibly to show that $y = x$ is not a line of invariant points)	If a particular point (eg (1, 1)) is used then supporting statement required for M1 that eg this must therefore apply on the line through (0, 0) and (1, 1). Can use $\begin{pmatrix}1 & 0\\2 & -1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$ to deduce that $x = y$.
	(e)		det **C** = $1\times-1-2\times0=-1$ So area of N is the same as area of M oe But the orientation of N is the reverse of the orientation of M.	B1ft B1ft B1ft [3]	1.1 2.2a 2.2a	Do not award if implication of statement is that area of N is negative. ft for det **C** < 0.	Allow "orientation changed"

Question			Answer	Marks	AO	Guidance	
3	(a)	(i)	**DR** 3(7 – 4i) – 4(7 + 4i) = 21 – 12i – 28 – 16i = –7 – 28i	**M1** **A1** **[2]**	1.1 1.1	z* correct and brackets opened. Allow sign mistake with -16i, but not z/z* mix-up	
	(a)	(ii)	**DR** $(7 – 4i + 1 – 3i)^2 = (8 – 7i)^2 = 64 – 112i + 49i^2$ 15 – 112i	**M1** **A1** **[2]**	1.1 1.1	Simplifying bracket and three term expansion. Allow z z* mix-up here	
	(a)	(iii)	**DR** $\frac{z+1}{z-1} = \frac{7-4i+1}{7-4i-1} = \frac{8-4i}{6-4i} \times \frac{6+4i}{6+4i}$ $= \frac{48+32i-24i+16}{36+16} = \frac{64}{52} + \frac{8}{52}i$	**M1** **A1** **[2]**	1.1 1.1	Simplifying and correct process for "realising" the denominator Allow $\frac{64+8i}{52} = \frac{16+2i}{13}$	Allow M1 if $\frac{z^*+1}{z^*-1}$ used correctly
	(b)		**DR** $\sqrt{(7^2 + (-4)^2)} = \sqrt{65}$ $\tan^{-1}\left(\pm\frac{4}{7}\right)$ $\sqrt{65}$ (cos(–0.519) + isin(–0.519))	**B1** **M1** **A1** **[3]**	1.1 1.1 2.5	Allow $\tan^{-1}\left(\frac{7}{4}\right)$ if it is clear that this being used correctly (eg from diagram) to find the argument or $\sqrt{65}$ cis(–0.519) or [$\sqrt{65}$, –0.519] or $\sqrt{65}$ cis(5.76) etc	Must be in the correct form ie c + is not c – is. If using [r, θ] then square brackets must be seen.
	(c)		**DR** 3 0.5 – –0.519 awrt 1.02	**B1ft** **M1** **A1ft** **[3]**	1.1 1.1 1.1	$\sqrt{585} \div$their $\lvert z \rvert$ Use of $\arg(z_1z_2) = \arg z_1 + \arg z_2$ Must be seen 0.5 – their arg(z) in [0, π/2]	

Question			Answer	Marks	AO	Guidance	
4	(a)		$\begin{pmatrix} a^2 & -2 \\ 1 & b^2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$	**B1**	3.1a	Rewriting LHS in matrix form	
			$\begin{pmatrix} a^2 & -2 \\ 1 & b^2 \end{pmatrix}^{-1} = \frac{1}{a^2b^2+2}\begin{pmatrix} b^2 & 2 \\ -1 & a^2 \end{pmatrix}$	**M1**	1.1	correct process for finding the inverse	
			$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{a^2b^2+2}\begin{pmatrix} b^2 & 2 \\ -1 & a^2 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$	**M1**	1.1	Multiplication by their inverse	
			$x = \frac{b^2+6}{a^2b^2+2},\ y = \frac{3a^2-1}{a^2b^2+2}$	**A1** **[4]**	1.1	Need to see $x = \ldots, y = \ldots$	
	(b)		Since $a^2b^2 = (ab)^2 \geq 0$ then $a^2b^2 + 2 > 0$ for all values of a and b the determinant of the matrix cannot be 0 (so the matrix is never singular)	**B1**	2.4	Argument must be complete and correct. eg $a^2b^2 + 2 \geq 0$ is B0.	
			so the inverse always exists and the method always works.	**B1** **[2]**	2.4		

Question			Answer	Marks	AO	Guidance	
5			$\alpha+\beta+\gamma=-\frac{3}{5},\ \alpha\beta+\beta\gamma+\gamma\alpha=-\frac{4}{5},\ \alpha\beta\gamma=-\frac{7}{5}$	B1	1.1a	For at least 2 correct	
			$A+B+\Gamma=2(\alpha+\beta+\gamma)=-\frac{6}{5}$	B1ft	2.1	2× their $\Sigma\alpha$	A, B and Γ are the roots of the new equation
			$AB+B\Gamma+\Gamma A=(\alpha+\beta)(\beta+\gamma)+(\beta+\gamma)(\gamma+\alpha)+(\gamma+\alpha)(\alpha+\beta)=3(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha^2+\beta^2+\gamma^2$	M1	2.1	Attempting to find the new $\Sigma\alpha\beta$ and expanding to a symmetrical form	
			$=\alpha\beta+\beta\gamma+\gamma\alpha+(\alpha+\beta+\gamma)^2=-\frac{11}{25}$	A1	1.1		
			$AB\Gamma=(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)=2\alpha\beta\gamma+\alpha(\alpha\beta+\beta\gamma+\gamma\alpha)-\alpha\beta\gamma+\beta(\alpha\beta+\beta\gamma+\gamma\alpha)-\alpha\beta\gamma+\gamma(\alpha\beta+\beta\gamma+\gamma\alpha)-\alpha\beta\gamma=(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha)-\alpha\beta\gamma$	M1	2.1	Opening brackets convincingly and writing in symmetrical form	Other correct forms are possible eg $\alpha^2\beta+\alpha^2\gamma+\beta^2\alpha+\beta^2\gamma+\gamma^2\alpha+\gamma^2\beta+2\alpha\beta\gamma$
			$=-\frac{3}{5}\times-\frac{4}{5}--\frac{7}{5}=\frac{12}{25}+\frac{35}{25}=\frac{47}{25}$	A1	1.1		
			$a=25 \Rightarrow 25x^3+30x^2-11x-47=0$	A1	1.1	Or any non-zero integer multiple	Must be integer coefficients

			Alternative method				
			Substitution $x=-\frac{3}{5}-u$	**B1***		Used or stated	
			$\alpha+\beta+\gamma=-\frac{3}{5},\ \left[\alpha\beta+\beta\gamma+\gamma\alpha=-\frac{4}{5},\ \alpha\beta\gamma=-\frac{7}{5}\right]$	**B1dep***		Might occur before the first B1 in candidates working	Note only the sum of roots needed here
			When $x=\alpha$, $u = -\frac{3}{5}-\alpha$ $= \alpha+\beta+\gamma-\alpha$ $= \alpha+\beta$	**B1****		Show that when x is one of the roots of the original equation, u is one of the roots of the new equation	
			When $x=\beta/\gamma$, $u = -\frac{3}{5}-\beta/\gamma$ $= \alpha+\beta+\gamma-\beta/\gamma$ $= \alpha+\gamma/\beta$	**B1 dep****		Show that $x=-\frac{3}{5}-u$ gives the other two required roots	
			$5\left(-\frac{3}{5}-u\right)^3+3\left(-\frac{3}{5}-u\right)^2-4\left(-\frac{3}{5}-u\right)+7=0$ $-5\left(u^3+3u^2\cdot\frac{3}{5}+3u\cdot\left(\frac{3}{5}\right)^2+\left(\frac{3}{5}\right)^3\right)$ $+3\left(u^2+2u\cdot\frac{3}{5}+\left(\frac{3}{5}\right)^2\right)+4\left(u+\frac{3}{5}\right)+7=0$	**M1**		Use substitution $x=-\frac{3}{5}-u$ and expand $\left(-\frac{3}{5}-u\right)^3$ and $\left(-\frac{3}{5}-u\right)^2$	$\left(-\frac{3}{5}-u\right)^3=-\left(u^3+\frac{9}{5}u^2+\frac{27}{25}u+\frac{27}{125}\right)$
			$-5u^3-6u^2+\frac{11}{5}u+\frac{47}{5}[=0]$	**A1**			
			$25u^3+30u^2-11u-47=0$	**A1**		Or any non-zero integer multiple	Must be integer coefficients. Must have "=0"
				[7]			

Question			Answer	Marks	AO	Guidance	
6			If $n = 9$, LHS = 9! = 362880 RHS = 2^{18} = 262144 < LHS [So true for $n = 9$]	B1	2.1	Basis case. Comparison must be explicit and correct	Bod sight of "true for n=1"
			Assume that $k! > 2^{2k}$ for some $k \geq 9$.	M1	2.1	Inductive hypothesis set up	Might see $2^{2k} = 4^k$ throughout
			$(k + 1)! = (k + 1)k! > (k + 1)\times 2^{2k}$ ...	M1	1.1	Considering for $k + 1$ and using inductive hypothesis correctly	Allow M1 for use of inductive $n = k$ step when showing $P_{k+1} \rightarrow P_k$
			... $> 9\times 2^{2k} > 4\times 2^{2k} = 2^2\times 2^{2k} = 2^{2+2k} = 2^{2(k+1)}$ ie $(k + 1)! > 2^{2(k+1)}$	A1	2.2a	Showing enough working to establish statement for $k + 1$	Might see ≥ 10 or ≥ 9 for > 9. A0 here if $k > 9$ stated earlier Do not allow if implication shown in the direction $P_{k+1} \rightarrow P_k$
			So true for $n = k \Rightarrow$ true for $n = k + 1$. But true for $n = 9$. So true for all integers $n \geq 9$	A1 [5]	2.4	Clear and complete conclusion	

Question			Answer	Marks	AO	Guidance	
7	(a)		$\mathbf{b}=\begin{pmatrix}2\\2\\1\end{pmatrix}\times\begin{pmatrix}-3\\1\\-1\end{pmatrix}$	M1	1.1a	Cross product either way round. Allow inclusion of λ and/or μ for M1 only.	
			$\mathbf{b}=\begin{pmatrix}-3\\-1\\8\end{pmatrix}$	A1	1.1	BC or any non-zero numerical multiple.	
			Alternative method $\mathbf{b}=\begin{pmatrix}p\\q\\r\end{pmatrix}$ and $\mathbf{b}.\begin{pmatrix}2\\2\\1\end{pmatrix}=0$ and $\mathbf{b}.\begin{pmatrix}-3\\1\\-1\end{pmatrix}=0$	M1		p, q or r could be any non-zero number	
			eg $p = 1$ and $2q + r = -2$ and $q - r = 3$ leading to $\mathbf{b}=\frac{1}{3}\begin{pmatrix}3\\1\\-8\end{pmatrix}$	A1			
				[2]			
	(b)		Since lines intersect $\begin{pmatrix}-12\\a\\-1\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\1\end{pmatrix}=\begin{pmatrix}2\\0\\5\end{pmatrix}+\mu\begin{pmatrix}-3\\1\\-1\end{pmatrix}$ for some λ and μ	M1	1.1		
			$\therefore\begin{pmatrix}-12\\a\\-1\end{pmatrix}.\mathbf{b}+\lambda\begin{pmatrix}2\\2\\1\end{pmatrix}.\mathbf{b}=\begin{pmatrix}2\\0\\5\end{pmatrix}.\mathbf{b}+\mu\begin{pmatrix}-3\\1\\-1\end{pmatrix}.\mathbf{b}$ but				
			$\begin{pmatrix}2\\2\\1\end{pmatrix}.\mathbf{b}=\begin{pmatrix}-3\\1\\-1\end{pmatrix}.\mathbf{b}=0\Rightarrow\mathbf{b}.\begin{pmatrix}-12\\a\\1\end{pmatrix}=\mathbf{b}.\begin{pmatrix}2\\0\\5\end{pmatrix}$	A1	1.1	AG	

			Alternative method Find where the two lines meet and obtain *a* = -6 (as below) $\begin{pmatrix}-3\\-1\\8\end{pmatrix}.\begin{pmatrix}-12\\a\\-1\end{pmatrix}$ and $\begin{pmatrix}-3\\-1\\8\end{pmatrix}.\begin{pmatrix}2\\0\\5\end{pmatrix}$ $\Rightarrow 36-a-8$ and $-6+40$ 36 - -6 -8 = 34 = -6 + 40 So dot products are equal	M1 A1		This can be awarded 2 marks in part c as long as some comment is made in part c (such as "a=-6" or "see above") Correct formulation of both their dot products using their **b** Conclusion needed	If using this method in part b then the intersection of the lines AND formulation of the dot products needs to happen for M1
				[2]			
	(c)		$\begin{pmatrix}-3\\-1\\8\end{pmatrix}.\begin{pmatrix}-12\\a\\-1\end{pmatrix}=\begin{pmatrix}-3\\-1\\8\end{pmatrix}.\begin{pmatrix}2\\0\\5\end{pmatrix}\Rightarrow 36-a-8=-6+40$ $a=-6$	M1ft A1 ft	1.1a 1.1	Correct formation of both dot products using their **b**	Stating *a*=-6 fine for 2 marks Also allow "see above" No marks if blank, even if a = -6 seen in (b)
			Alternative method $-12+2\lambda=2-3\mu$ & $-1+\lambda=5-\mu$ $\Rightarrow\lambda=4$, $\mu=2$ $a=\mu-2\lambda=-6$	M1ft A1 ft		Forming *x* and *z* equations using their **b** and solving for λ and μ From *y* equation	
				[2]			

Question			Answer	Marks	AO	Guidance	
8	(a)		C_1 is (represented by) the line $x = 2d^2 + 18$	**B1**	3.1a	Seen or implied in solution	
			C_2 is (represented by) the (half-)line $y = x + c$	**M1**	3.1a	For understanding the C_2 is a line or half-line whose gradient is 1.	
			$3 = 12d + c$	**M1**	3.1a	Complete line would pass through the point $(12d, 3)$ or $12d + 3i$.	Half line starting at $(12d, c)$ and with angle $\frac{\pi}{4}$
			$y = x + 3 - 12d$	**A1**	1.1		
			When $x = 2d^2 + 18$, $y = 2d^2 - 12d + 21$	**M1**	1.1	Attempt at y coordinate	
			$2d^2 + 18 + (2d^2 - 12d + 21)i$	**A1**	3.2a	Must be in complex number form	or eg $2(d^2 + 9) + (2(d - 3)^2 + 3)i$
			Alternative Method				
			C_1 is (represented by) the line $x = 2d^2 + 18$	**B1**		SOI	
			C_2 is (represented by) the (half-)line starting at the point $12d + 3i$	**M1**			
			C_2 half line has gradient 1	**M1**		Correct half line needed here	Could be shown by making angle $\frac{\pi}{4}$ with positive x direction
			Right-angled triangle indicated with base length $(2d^2 + 18) - 12d$	**M1**		Follow through C_1 line and start of C_2 half line	
			y coordinate at $3 + ((2d^2 + 18) - 12d)$	**M1**		Attempt at y coordinate using base of triangle = height of triangle and adding on 3i	
			POI at $2d^2 + 18 + (2d^2 - 12d + 21)i$	**A1**		Must be in complex number form	or eg $2(d^2 + 9) + (2(d - 3)^2 + 3)i$
				[6]			

	(b)		When $d = 3$, the PoI would be 36 + 3i and $C_2 = \left\{ z : \arg\left(z - (36 + 3i)\right) = \frac{1}{4}\pi \right\}$ But 36 + 3i is not in C_2 since arg0 is not defined	**M1** **A1**	3.1a 3.2b	Both AG. Or arg0 is not $\pi/4$	NB $C_1 \cap C_2 = \varnothing$ without justification is M0A0.
			Alternative Method For the intersection to exist we need $2d^2 + 18 > 12d$ $d^2 - 6d + 9 > 0$ $(d-3)^2 > 0$ $[d \neq 3]$ Hence d cannot be equal to 3	**M1** **A1**		Set up inequality using x coordinates of vertical line and starting point of half line Or "$d = 3$" is not valid etc.	
				[2]			

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