Coordinate Geometry (2 dimensions only): Further Maths

The straight line

→ gradient: a number that describes both the direction and steepness of the line.

• To find the gradient: change in y ÷ change in x of 2 points on the line.

→ Parallel and perpendicular lines

• Parallel lines have the same gradient.

• Perpendicular lines have the negative reciprocal gradient (eg $\frac{1}{2} \Rightarrow -2$)

• To prove/disprove a triangle has a right-angle from 3 given points:

1. Optional: draw a sketch of the triangle with the given information.
2. Using the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, find the length of each of the triangle's 3 sides (the distance between each of the 3 points).
3. Use Pythagoras' Theorem ($a^2 + b^2 = c^2$) to prove/disprove the longer side is/isn't equal to the 2 shorter sides.
4. Give a statement that states the triangle is/isn't a right-angled triangle.

e.g "Show that A (13, −1), B (−9, 3) and C (−3, −9) forms a right-angled triangle."

$d = \sqrt{(-9-(-3))^2 + (3-(-9))^2}$ $\quad$ $d = \sqrt{(13-(-9))^2 + (-1-3)^2}$ $\quad$ $d = \sqrt{(-9-(-1))^2 + (-3-13)^2}$

$d = \sqrt{36+144}$ $\quad$ $d = \sqrt{484+16}$ $\quad$ $d = \sqrt{64+256}$

$d = \sqrt{180}$ $\quad$ $d = \sqrt{500}$ $\quad$ $d = \sqrt{320}$

$a^2 + b^2 = c^2$

$(\sqrt{180})^2 + (\sqrt{320})^2 = (\sqrt{500})^2$

$180 + 320 = 500$

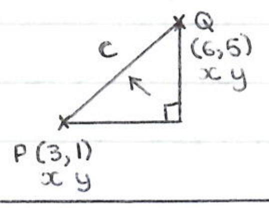
$500 = 500$ ✓ "Yes, it is a right-angled triangle because it works with Pythagoras Theorem which only works for right-angled triangles."

• You can also find the gradient of the sides & see if any are perpendicular to each other to check for right angles.

→ Length of a line segment

1. Sketch a graph with the given information & join together the points to make a right-angled triangle.
2. Work out the differences between x and y of the two points & label the triangle's sides.
3. Use Pythagoras Theorem ($a^2 + b^2 = c^2$) to work out the length of the segment.

"Work out the distance between P = (3, 1) & Q = (6, 5)."



$3^2 + 4^2 = c^2$

$25 = c^2$

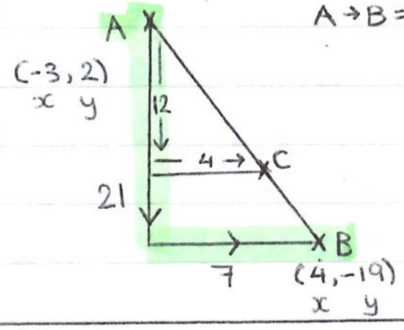
$5 = c$

→ Dividing line segments

• To divide a line segment with a ratio to find a point on a line:

1. Draw a sketch of all the given information.
2. Workout what 1 part of the ratio does. ($\frac{1}{x} = \updownarrow y \longleftrightarrow z$).
3. Multiply to find where the point is in relation to the others & workout from the co-ordinate it is coming from.

"AB is a straight line A = (-3,2) B = (4,-19) C is a point on AB such that AC : AB = 4 : 3. Find the co-ordinates of C."



$A \to B = \frac{7}{7} \downarrow 21 \to 7$

$\frac{1}{7} = \downarrow 3 \to 1$

$\frac{4}{7} = \downarrow 12 \to 4$

$C = (1, -10)$

TIP! Look at the ratio to find out the direction you're moving from & where the missing point is.

- To find the midpoint of the line segment: add the x/y coordinates & half the result. Formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

→ The equation of a straight line

- Standard: $y = mx + c$, where 'm' is the gradient & 'c' is the y-intercept. It is useful for when you know the y-intercept and the gradient.
- Point-slope form: $y - y_1 = m(x - x_1)$, where 'm' is the gradient and x_1 & y_1 are the coordinates of a point on the line (x_1, y_1). It is useful for when you know a point on the line and the gradient.
- The formula can be rearranged into different forms.

eg "Find the equation of the line with the point (1,3) & a gradient of -2 in the form:" "$y - y_1 = m(x - x_1)$"

$y - 3 = -2(x - 1)$

"$y = mx + c$"

By rearranging: $y - 3 = -2x + 2$ (expand brackets of other form)

$y = -2x + 5$ (+3)

By substitution into the formula: $y = mx + c$

$(3) = (-2)(1) + c$

$5 = c$ (+2)

$\therefore y = -2x + 5$

→ Drawing straight lines

general method! • Table method: find 3 points by subbing in the x-coordinate to get the y-coordinate.

"$y = x + 2$"

x	0	2	4
y	2	4	6

for fractional gradients! • Equation method: plot the y-intercept & use the gradient to plot points by going

across units $\frac{y \text{ — go up or down}}{x \text{ — go left or right}}$. (e.g gradient 2 = two units right per one unit up)
(eg 2. gradient $3/4$ = 3 units right per 4 units up).

• $x=0, y=0$ method: sub in the values $x=0$ & $y=0$, then find their counterpart co-ordinate for the x and y intercepts.

The coordinate geometry of circles

→ The circle equation

general: $(x-a)^2+(y-b)^2=r^2$, where (a,b) is the centre & 'r' is the radius.

egs $x^2+y^2=r^2$ → centre = (0,0)

$(x-3)^2+(y+2)^2=25$ → centre = (3,-2) radius = 5

→ Circle geometry

• Circle theorems used in circle geometry:

→ the angle in a semi-circle is 90°

→ the angle between tangent and radius is 90°

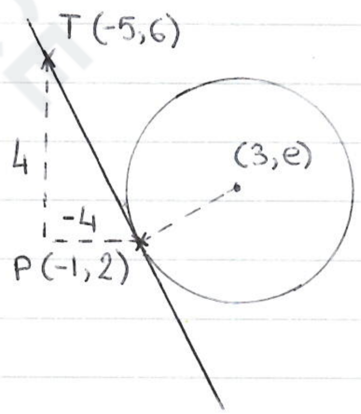
→ the perpendicular from the centre to a chord bisects the chord

→ tangents from an external point are equal in length

• To find the equation of a tangent at a point on a circle:

1. Find the gradient of the radius of the circle.
2. Find the gradient of the tangent by doing the negative reciprocal of the radius' gradient. (The radius is perpendicular to the tangent of the circle at a point.)
3. To write the equation in standard form ($y=mx+c$): substitute a point on the tangent to find the y-intercept, then write out the tangent's equation.
   To write the equation in point-slope form ($y-y_1=m(x-x_1)$): include a point on the tangent into the equation (in place of x_1 & y_1) in the tangent's equation.

eg "A circle has a centre of (3,e). PT is a tangent to the circle at P. P = (-1,2) & T = (-5,6). Find the value of e."



→ gradient of the tangent = change in y ÷ change in x
$= 4 \div -4 = -1$

→ the tangent is perpendicular to the radius.

→ gradient of the radius = 1 (negative reciprocal of -1)

→ radius equation: $y-2=x+1$

sub in $(3,e)$ → $e-2=3+1$

$e=6$ (+2)