# Differentiation - Tangents, normals and gradients

Name: ________________________________

Class: ________________________________

Date: ________________________________

**Time:**

**Total marks available:**

**Total marks achieved:** ______

A Level Mathematics : Pure Mathematics

Subject: Mathematics

Topic 7 : Differentiation - Tangents, normals and gradients

Exam Papers Practice

Type: Topic Questions

To be used by all students preparing for Edexcel A Level Mathematics - Students of other Boards may also find this useful

**In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

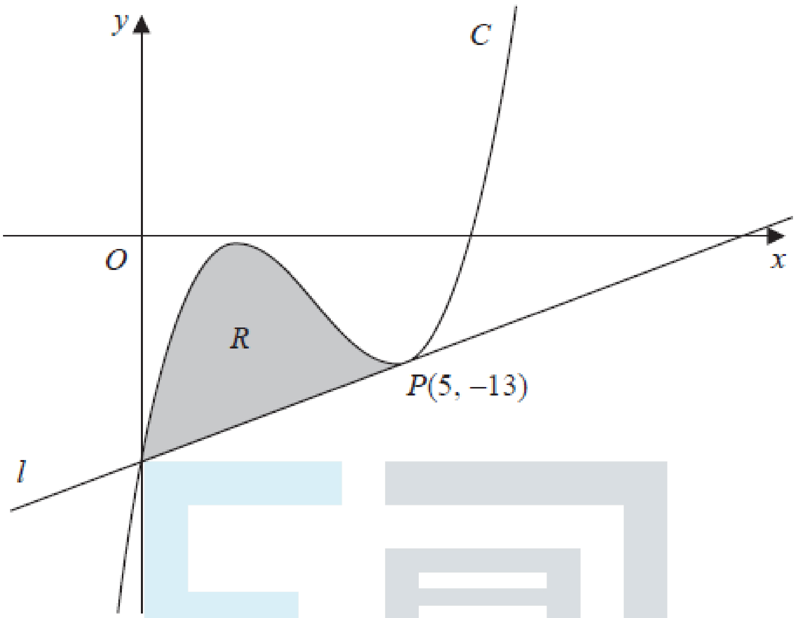


**Figure 2**

Figure 2 shows a sketch of part of the curve *C* with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point *P*(5, –13) lies on *C*

The line *l* is the tangent to *C* at *P*

(a) Use differentiation to find the equation of *l*, giving your answer in the form $y = mx + c$ where *m* and *c* are integers to be found.

**(4)**

(b) Hence verify that *l* meets *C* again on the *y*-axis.

**(1)**

The finite region *R*, shown shaded in Figure 2, is bounded by the curve *C* and the line *l*.

(c) Use algebraic integration to find the exact area of *R*.

**(4)**

**(Total for question = 9 marks)**

Q2.

The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of *C*,

(ii) the exact radius of *C*, giving your answer as a simplified surd.

**(4)**

The line *l* has equation $y = 3x + k$ where $k$ is a constant.

Given that *l* is a tangent to *C*,

(b) find the possible values of $k$, giving your answers as simplified surds.

**(5)**

**(Total for question = 9 marks)**

Q3.

The curve *C* has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

**(3)**

(b) Verify that *C* has a stationary point when $x = 2$

**(2)**

(c) Determine the nature of this stationary point, giving a reason for your answer.

**(2)**

**(Total for question = 7 marks)**

Q4.

The curve $C$ has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of $t$.

**(2)**

The point $P$ lies on $C$ where $t = \frac{2\pi}{3}$

The line $l$ is the normal to $C$ at $P$.

(b) Show that an equation for $l$ is

$$2x - 2\sqrt{3}y - 1 = 0$$

**(5)**

The line $l$ intersects the curve $C$ again at the point $Q$.

(c) Find the exact coordinates of $Q$.

You must show clearly how you obtained your answers.

**(6)**

**(Total for question = 13 marks)**

Q5.

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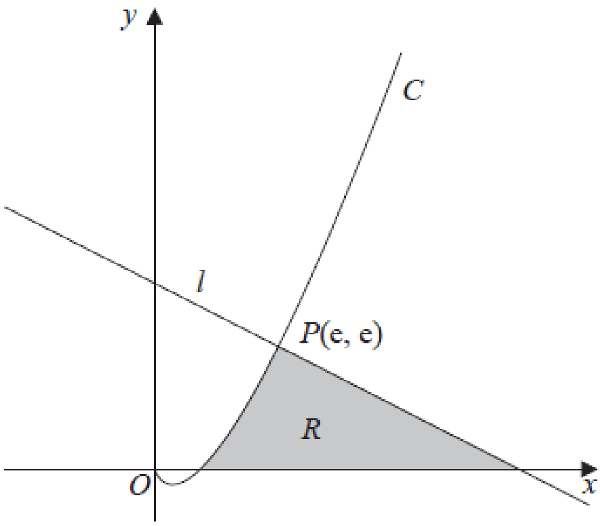


**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with equation $y = x \ln x, \quad x > 0$

The line $l$ is the normal to $C$ at the point $P(\mathrm{e}, \mathrm{e})$

The region $R$, shown shaded in Figure 2, is bounded by the curve $C$, the line $l$ and the $x$-axis.

Show that the exact area of $R$ is $A\mathrm{e}^2 + B$ where $A$ and $B$ are rational numbers to be found.

**(10)**

**(Total for question = 10 marks)**

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Q6.

$$\frac{1 + 11x - 6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$$

(a) Find the values of the constants $A$, $B$ and $C$.

**(4)**

$$\mathrm{f}(x) = \frac{1 + 11x - 6x^2}{(x-3)(1-2x)} \qquad x > 3$$

(b) Prove that $\mathrm{f}(x)$ is a decreasing function.

**(3)**

**(Total for question = 7 marks)**

Q7.

The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where $k$ is a constant.

(a) Deduce the value of $k$.

**(1)**

(b) Prove that

$$g'(x) > 0$$

for all values of $x$ in the domain of g.

**(3)**

(c) Find the range of values of $a$ for which

$$g(a) > 0$$

**(2)**

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**(Total for question = 6 marks)**

Q8.

The curve $C$ has equation

$$px^3 + qxy + 3y^2 = 26$$

where $p$ and $q$ are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where $a$, $b$ and $c$ are integers to be found.

**(4)**

Given that

- the point *P* (–1, – 4) lies on *C*
- the normal to *C* at *P* has equation 19*x* + 26*y* + 123 = 0

(b) find the value of *p* and the value of *q*.

**(5)**

**(Total for question = 9 marks)**

Q9.

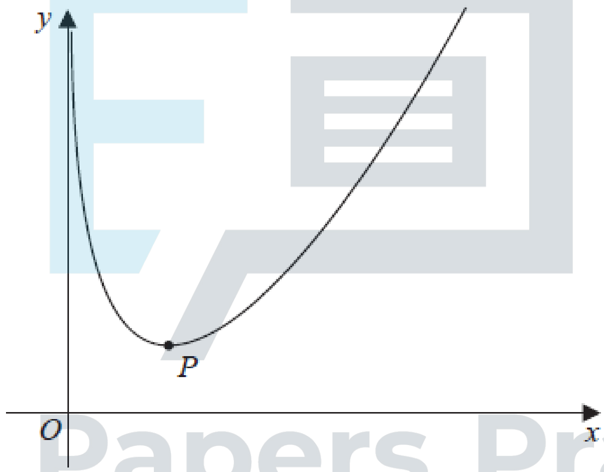


**Figure 1**

Figure 1 shows a sketch of the curve *C* with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

**(4)**

The point *P*, shown in Figure 1, is the minimum turning point on *C*.

(b) Show that the *x* coordinate of *P* is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

**(3)**

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of $x_2$ to 5 decimal places,

(ii) the $x$ coordinate of $P$ to 5 decimal places.

**(3)**

**(Total for question = 10 marks)**

Q10.

The curve $C$ has parametric equations

$$x = \sin 2\theta \qquad y = \text{cosec}^3\theta \qquad 0 < \theta < \frac{\pi}{2}$$

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(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $\theta$

**(3)**

(b) Hence find the exact value of the gradient of the tangent to $C$ at the point where $y = 8$

**(3)**

**(Total for question = 6 marks)**

Q11.

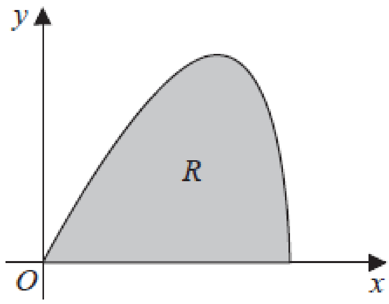


**Figure 3**

The curve shown in Figure 3 has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region $R$, shown shaded in Figure 3, is bounded by the curve and the $x$-axis.

(a) (i) Show that the area of $R$ is given by $\displaystyle\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\,dt$

**(3)**

(ii) Hence show, by algebraic integration, that the area of $R$ is exactly 20

**(3)**

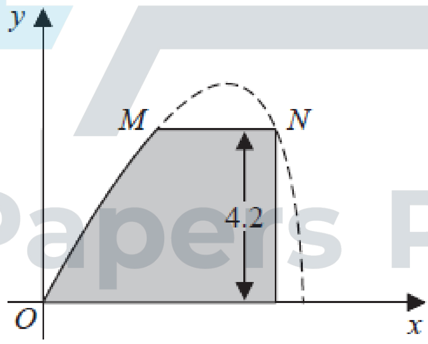


**Figure 4**

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4.
Using the model and given that

- $x$ and $y$ are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width $MN$ along the top of the dam

(b) calculate the width of the walkway.

**(5)**

**(Total for question = 11 marks)**

Q12.

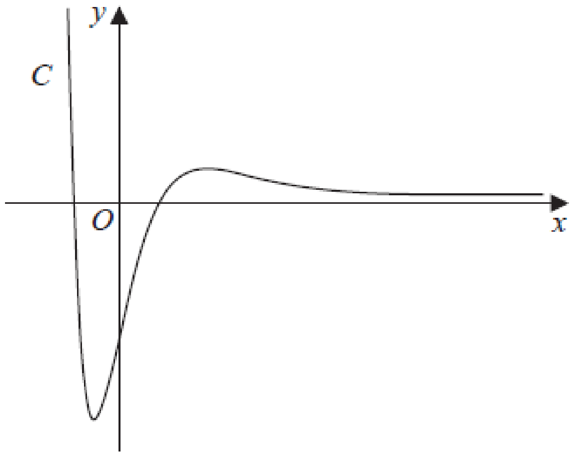


**Figure 2**

Figure 2 shows a sketch of the curve $C$ with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4(x^2 - 2)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8(2 + x - x^2)\mathrm{e}^{-2x}$

**(3)**

(b) Hence find, in simplest form, the exact coordinates of the stationary points of $C$.

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**(3)**

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

**(3)**

**(Total for question = 9 marks)**

Q13.

(a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int_p^q \frac{6\,\mathrm{d}u}{u(3+2u)}$$

where $p$ and $q$ are positive constants to be found.

**(4)**

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \ln a$$

where $a$ is a rational constant to be found.

**(6)**

**(Total for question = 10 marks)**

Q14.

A large spherical balloon is deflating.

At time $t$ seconds the balloon has radius $r$ cm and volume $V$ cm$^3$

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where $k$ is a positive constant.

**(3)**

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

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(b) solve the differential equation to find a complete equation linking $r$ and $t$.

**(5)**

(c) Find the limitation on the values of $t$ for which the equation in part (b) is valid.

**(2)**

**(Total for question = 10 marks)**

Q15.

The curve $C$ has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

**(4)**

(b) Prove that $C$ has a point of inflection at $x = \sqrt[4]{27}$

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**(3)**

**(Total for question = 7 marks)**