6.5 Boolean algebra

Name: _____

Class: _____

Date: _____

Time: **183 minutes**

Marks: **168 marks**

Comments:

Q1.

Represent the Boolean equation $\overline{A} + \overline{B} \cdot C$ as a logic circuit by drawing a diagram of it in the space below.

(Total 3 marks)

Q2.

Using the rules of Boolean algebra, simplify the following expression.

$$\overline{\overline{A \cdot (\overline{B} + 0)} \cdot \overline{\overline{A} \cdot (B + B)}}$$

You **must** show your working.

Answer _____

(Total 4 marks)

Q3.

Using the laws of Boolean algebra, show that:

$$(A + B) \cdot (B + C \cdot (D + \overline{D})) = A \cdot C + B$$

You **must** show your working.

(Total 4 marks)

Q4.

Using the laws of Boolean algebra, simplify the following Boolean expression.

$$(X + Y) \bullet (X + \overline{Y})$$

You **must** show your working.

Answer _____

(Total 4 marks)

Q5.

A computer process, X, can only start executing once processes A and B have finished

executing and either communication channel C or communication channel D or both are available to use.

The states of processes and communication channels can be read using the following Boolean variables:

- **A** is set to TRUE if process A has completed and FALSE if process A is still running.
- **B** is set to TRUE if process B has completed and FALSE if process B is still running.
- **C** is set to TRUE if communication channel C is available and FALSE if it is not available.
- **D** is set to TRUE if communication channel D is available and FALSE if it is not available.

The Boolean variable **X** should be set to TRUE if the values of the variables **A**, **B**, **C** and **D** indicate that process X can start and to FALSE if they indicate that process X cannot start yet.

(a) Draw a logic circuit that will represent the logic of the system described above for the inputs **A**, **B**, **C** and **D** and the output X.

(3)

(b) Write a Boolean expression to represent the logic used to start process X.

X = _____

(2)

(Total 5 marks)

Q6.

Using the rules of Boolean algebra, simplify the following Boolean expression.

$$\overline{(\overline{A} + A \cdot (A + B))} + (\overline{B} \cdot \overline{C})$$

You **must** show your working.

_____

_____

Answer _____

(Total 4 marks)

Q7.

Simplify the following Boolean expression:

$$\overline{\overline{A} + \overline{B} + B.\overline{A}}$$

(Total 3 marks)

Q8.

Using the rules of Boolean algebra, simplify the following Boolean expression.

$$(\overline{A} + B).\overline{(A + (\overline{B + A}))}$$

You **must** show your working.

Answer _____

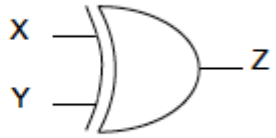
(Total 4 marks)

Q9.

(a) What is the name of the logic gate represented by the truth table and symbol shown in **Figure 1**?

Figure 1

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

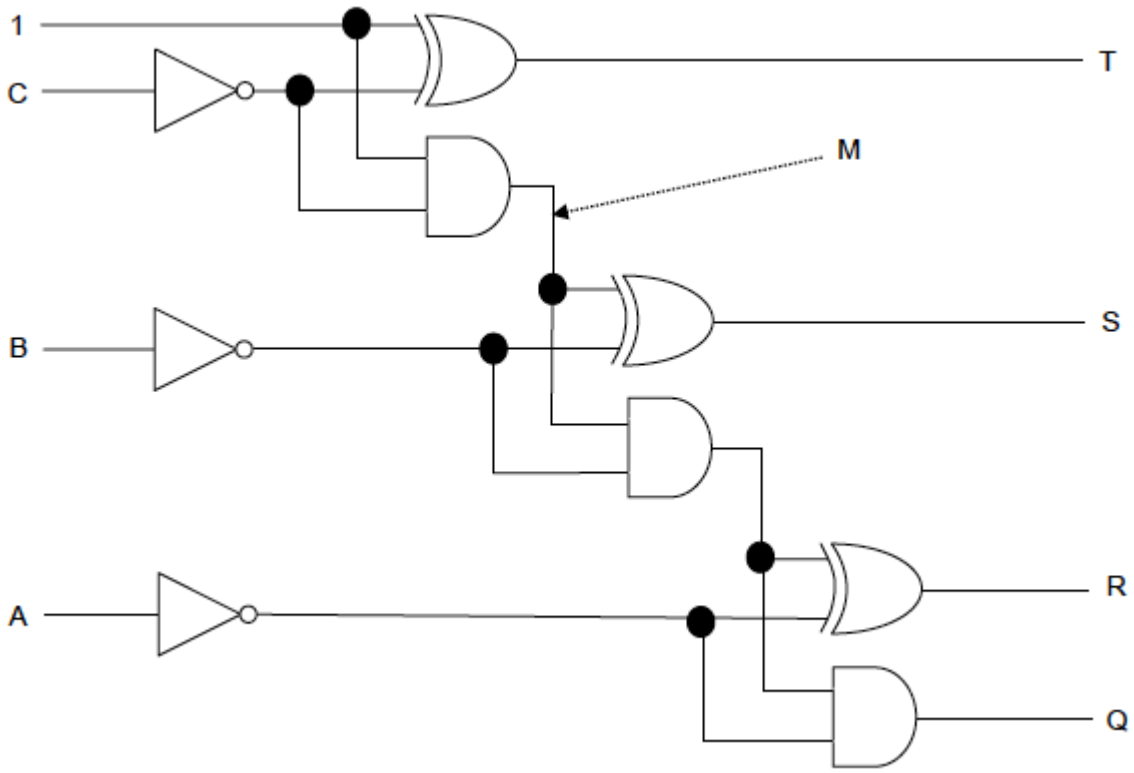


_____

(1)

Figure 2 shows a logic circuit that might be found inside a processor.

Figure 2



(b) The value at **M** (indicated by the arrow in Figure 2) is represented by the following Boolean expression.

$1 . \bar{C}$

Simplify this Boolean expression.

_____

_____

(1)

(c) The output **T** (in Figure 2) is represented by the following Boolean expression.

$1 \oplus \bar{C}$

Simplify this Boolean expression.

_____

_____

(1)

(d) Complete the missing cells in the table.

Four rows of the truth table for the circuit in Figure 2

Inputs			Outputs		
C	**B**	**A**	**T**	**S**	**R**
0	0	0	0	0	0

0	0	1			
0	1	0			
0	1	1	0	1	0

(3)

(e) The logic circuit shown in **Figure 2** obtains the two's complement of a 3-bit binary number. Explain how this circuit could be used by a processor when subtracting one 3-bit binary number from another.

(1)

(f) The circuit in **Figure 2** can be simplified so that it uses fewer logic gates but still has the same functionality. Changing the design of the circuits used in a processor can improve processor performance. Increasing the number of cores can also improve processor performance.

State **three** other factors that can improve processor performance. For each factor, explain how it will improve processor performance.

Factor: _____

How improves: _____

Factor: _____

How improves: _____

Factor: _____

How improves: _____

(6)

(Total 13 marks)

Q10.

(a) (i) State the name of the logic gate that is represented by the truth table below.

Inputs		Output
A	**B**	**Q**
0	0	0
0	1	0
1	0	0
1	1	1

(1)

(ii) State the name of the logic gate that is represented by the truth table below.

Inputs		Output
A	**B**	**Q**
0	0	1
0	1	0
1	0	0
1	1	0

(1)

(b) A fish tank has a controller that can monitor sensors and control external devices.

- Sensor A returns a 1 (true) when the water level is too low and a 0 (false) otherwise
- Sensor B returns a 1 (true) when the water level is too high and a 0 (false) otherwise
- Sensor C returns a 1 (true) when the water temperature is above 28°C and a 0 (false) otherwise
- Sensor D returns a 1 (true) when the water temperature is below 23°C and a 0 (false) otherwise.

The controller also has outputs to the following devices:

- The heater is turned on when output H is 1 (true) and turned off when output H is 0 (false)
- The top-up water pump is on when output J is 1 (true) and turned off when output J is 0 (false)

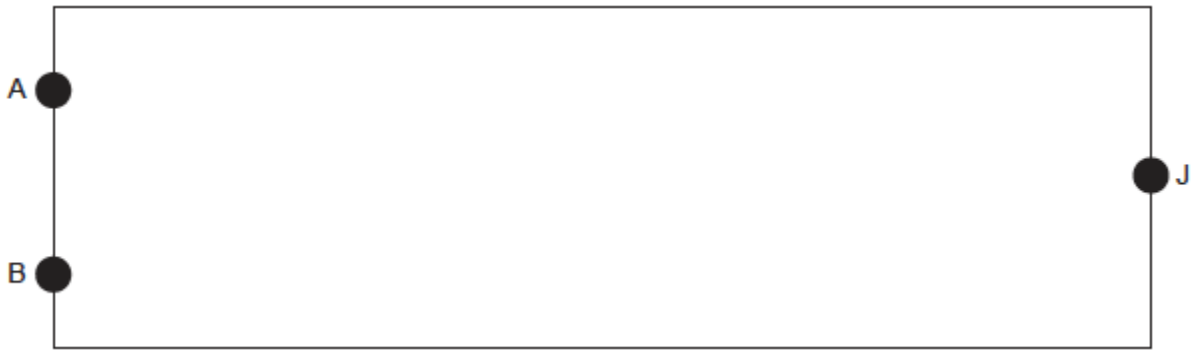
The controller can also read the current state of both H and J to see whether the devices are turned on or off.

(i) The top-up water pump is turned on and off according to the Boolean

equation:

$$J = A \cdot \overline{B}$$

Draw the logic circuit corresponding to this equation in the box below.



(2)

(ii) A fault alarm should sound when either of the following two conditions are met:

- if the water temperature is above 28°C and the heater is on
- if the water level is too low and the top-up water pump is **not** on.

Write a Boolean equation to represent the logic required to turn the fault alarm on when either or both of these conditions are met.

(3)

(c) **Without** using a truth table, simplify the following Boolean expression:

$$\overline{A \cdot B} + \overline{B + C}$$

(3)

(Total 10 marks)

Q11.

(a) Complete the table below and draw the symbol for an AND gate in the box.

Truth table for an AND gate

Input A	Input B	Output

AND gate symbol

(2)

(b) Using the laws of Boolean algebra, simplify the following Boolean expression.

A.B. (A + B)

Answer _____

(3)

(c) Using the laws of Boolean algebra, simplify the following Boolean expression.

$(X + Y).(X + \overline{Y})$

Answer _____

(3)

(Total 8 marks)

Q12.

A burglar alarm system is to be implemented that has the following sensors:

- a door sensor **D** that outputs TRUE when the door is open and FALSE when the door is shut

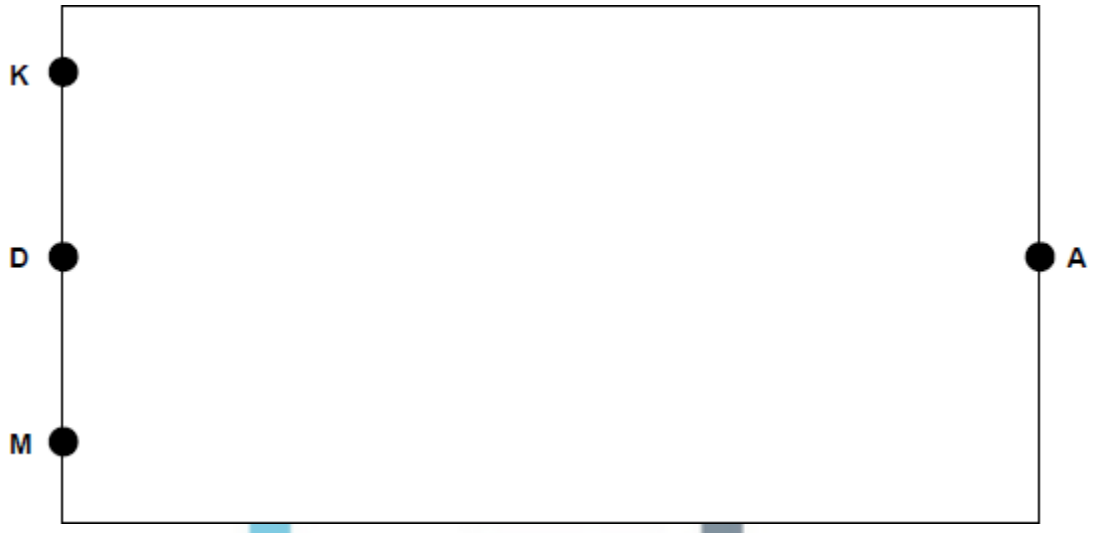
- a pressure mat sensor **M** that outputs TRUE while a weight is detected on it and FALSE when no weight is detected on it.

The alarm also has a key **K** that turns the alarm on and off. **K** outputs a TRUE signal when the alarm is switched on and FALSE when the alarm is off.

The alarm output **A** sounds a bell. It should be TRUE if:

- the alarm is on AND
- either of the sensors **D** or **M** are set to the value TRUE.

(a) Draw a logic circuit that will behave as described above for the inputs **D**, **M** and **K** and the output **A**.



(2)

(b) Write a Boolean expression to represent the logic of this alarm system.

A=_____

(2)

(c) In this alarm system, the alarm bell will sound only while the door is open or a weight is placed on the pressure mat. If someone who has stepped on to the mat moves off it, or an open door is closed, the alarm bell will stop ringing.

A D-type flip-flop could be incorporated into the logic circuit so that the alarm bell would continue to sound after a person closed the door or moved off the pressure mat.

Explain how this could be achieved. In your answer refer to:

- why a D-type flip-flop would be suitable for this task
- where the D-type flip-flop would need to be inserted into the circuit
- what additional input the D-type flip-flop would need.

_____

_____

_____

_____

(3)
(Total 7 marks)

Q13.

(a) Complete the truth table below for a NAND gate.

NAND gate		
Input A	**Input B**	**Output**
0	0	
0	1	
1	0	
1	1	

(1)

(b) Multiplexors are used in electronic switching.

A 2-to-1 multiplexor has a Boolean equation where A and B are two inputs, S is the selector input, and Q is the output.

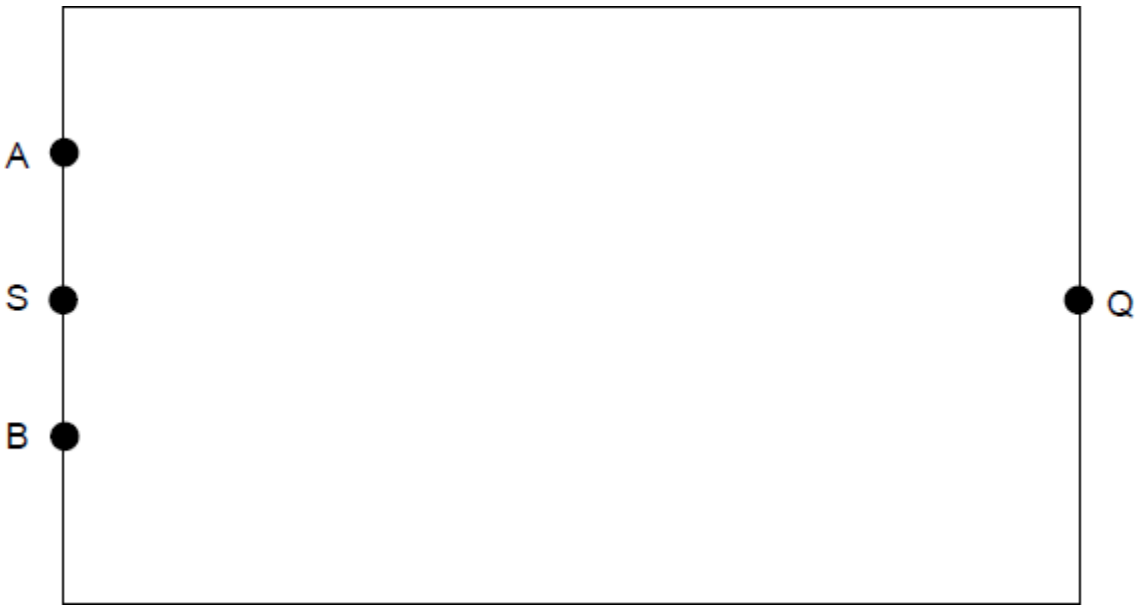
$$Q = (A.\overline{S}) + (B.S)$$

(i) Complete the truth table for the above Boolean equation.

S	A	B	$\overline{S}$	$A.\overline{S}$	B.S	Q
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

(3)

(ii) Draw a circuit for the Boolean equation in the rectangle below.



(4)

(iii) By considering its inputs and outputs, describe what the 2-to-1 multiplexor circuit does.

(1)

(Total 9 marks)

Q14.

(a) Complete the truth tables for the following logic gates.

OR gate

Input A	Input B	Output
0	0	
0	1	
1	0	
1	1	

NAND gate

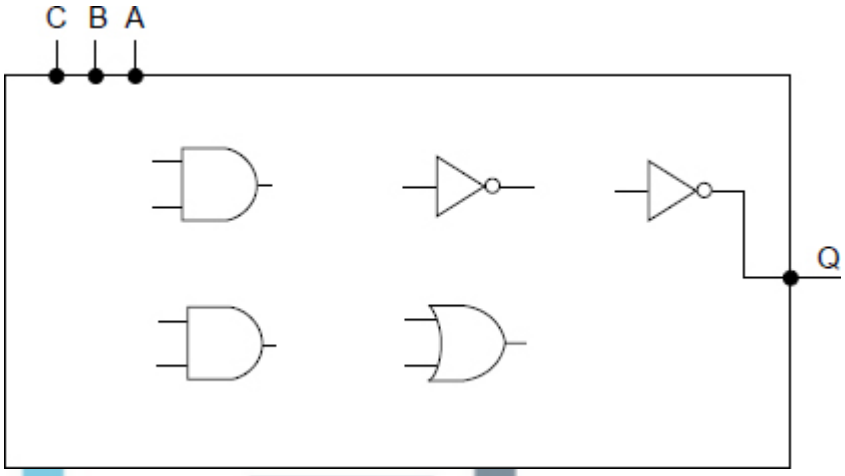
Input A	Input B	Output
0	0	
0	1	

1	0	
1	1	

(2)

(b) Represent the following Boolean equation as a logic circuit by completing the diagram below.

$$Q = \overline{\overline{A \cdot B} + B \cdot C}$$



(5)

(c) Simplify the following expression.

$$\overline{\overline{A} + \overline{B} + B \cdot \overline{A}}$$

Show each stage of your working.

(2)

Final answer _____

(1)

(Total 10 marks)

Q15.

(a) State the names of the logic gates represented by each of the three truth tables below.

Input A	Input B	Output
0	0	0
0	1	0

Logic gate name _____

1	0	0
1	1	1

Input A	Input B	Output
0	0	1
0	1	0
1	0	0
1	1	0

Logic gate name _____

Input A	Input B	Output
0	0	0
0	1	1
1	0	1
1	1	0

Logic gate name _____

(3)

(b) Simplify the following Boolean expressions.

(i) B . (A+$\overline{A}$)

_____

_____

(1)

(ii) A . B + B

_____

_____

(1)

(iii) $\overline{B} \cdot (\overline{\overline{A}+\overline{B}})$

_____

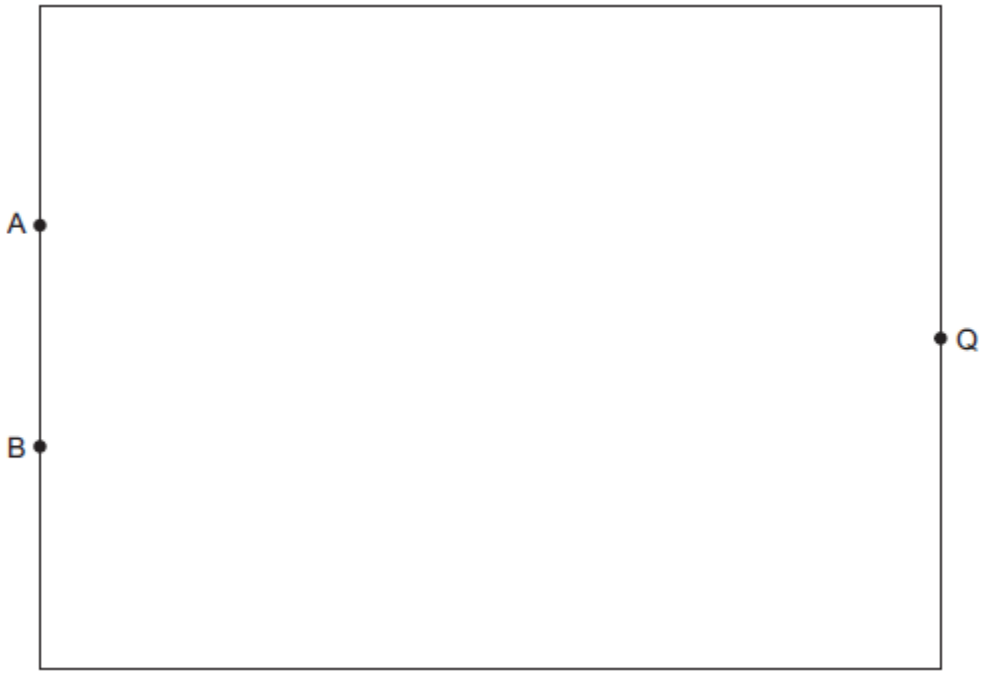
_____

(2)

(c) Draw a logic circuit for the following Boolean expression:

$$Q = (A \oplus B) \cdot B$$

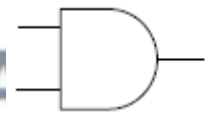
You will need to make use of the symbols below when drawing your logic circuit.



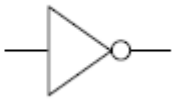
(2)
(Total 9 marks)

Q16.

(a) Represent the Boolean equation Q = $\overline{A}.\overline{B}$ as a logic circuit by drawing a diagram using **only** the following symbols:



AND



NOT

(3)

(b) Use the following truth tables to demonstrate that A + B = $\overline{\overline{A}.\overline{B}}$

A	B	A + B
0	0	
0	1	
1	0	
1	1	

A	B				$\overline{\overline{A}.\overline{B}}$
0	0				
0	1				
1	0				
1	1				

(4)

(c) What is the name commonly associated with the statement A + B = $\overline{\overline{A}.\overline{B}}$?

_____

(1)

(d) Simplify the Boolean expression below.

$$A.B.\overline{C} + A.\overline{C}$$

Show each stage of your workin.

(2)

Final answer _____

(1)

(Total 11 marks)

Q17.

(a) Complete the truth tables for the following logic gates.

AND Gate		
Input X	**Input Y**	**Input Q**
0	0	
0	1	
1	0	
1	1	

XOR Gate		
Output X	**Input Y**	**Output Q**
0	0	
0	1	
1	0	
1	1	

(2)

(b) A line-following robot has three sensors. It moves along a black line on a white background whilst the following conditions are met:

- the ultrasonic sensor U does not detect any obstacle
- either, but not both, of the infrared sensors L and R are on the black line.

Sensor U returns 1 if it detects an obstacle and 0 if the path is clear.
Sensors L and R each return 1 if they detect black and 0 if they detect white.

A logic circuit will process the input from the sensors and produce an output M.

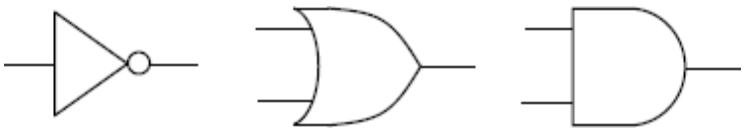
M should be 1 if the robot is to move and 0 if the robot should stop.

(i) Represent the output M as a Boolean expression.

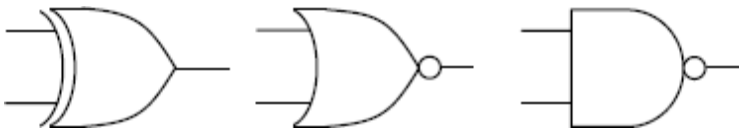
M = _____

(3)

(ii) The following symbols are used to represent logic gates:

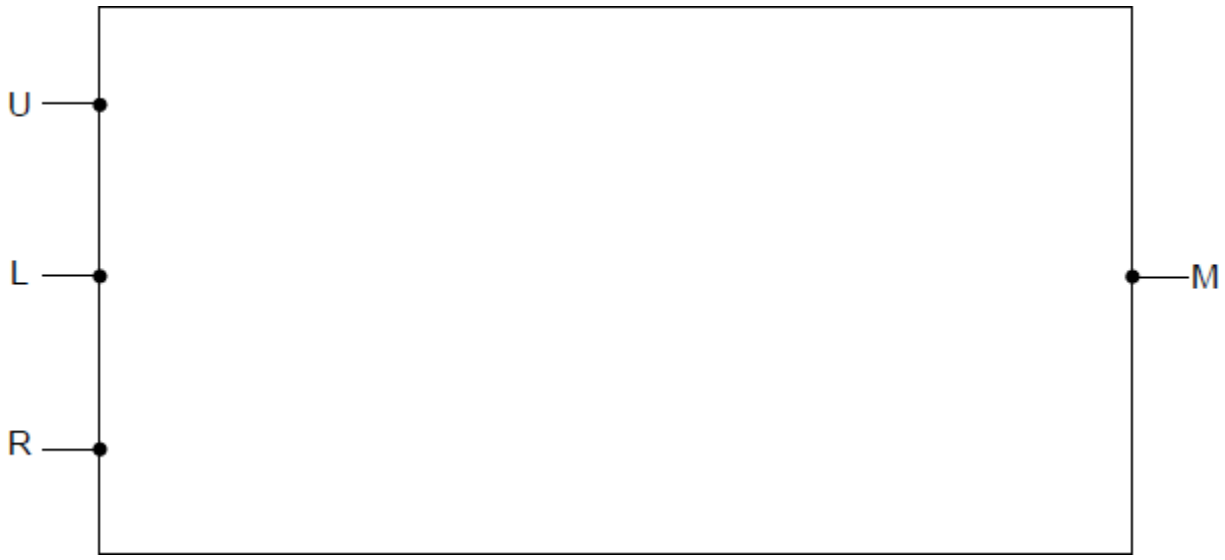


NOT OR AND



XOR NOR NAND

Using a combination of any of the above logic gates draw a logic circuit for this system in the box below. You will **not** need to use all of the different types of logic gates.



(3)

(c) Apply De Morgan's Law(s) to the following expression and simplify the result.

$Q = \overline{\overline{A} + (\overline{B} \cdot A)}$

Show the stages of your working.

_____

_____

_____

_____

(2)

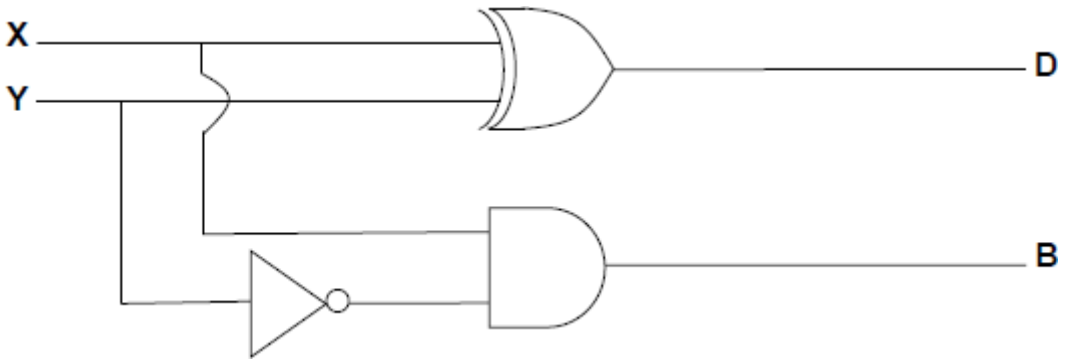
Final answer _____

(1)

(Total 11 marks)

Q18.

The diagram below shows a logic circuit.



(a) Write a Boolean expression for **D**.

_____

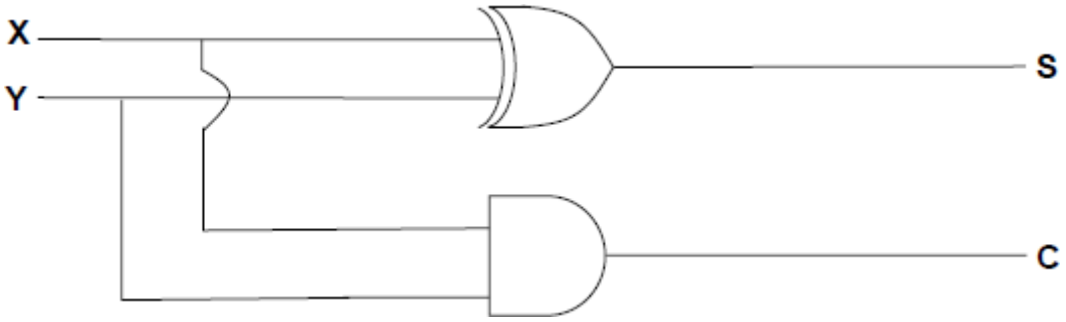
(1)

(b) Write a Boolean expression for **B**.

_____

(1)

(c) The diagram below shows a different logic circuit.



(i) Complete the truth table below for the logic circuit in the diagram above.

Inputs		Outputs	
X	**Y**	**C**	**S**
0	0		
0	1		
1	0		
1	1		

(2)

(ii) What arithmetic function does the logic circuit in the diagram above perform?

_____

(1)

(d) **Without** using a truth table, simplify the Boolean expression below.

$(X + Y) . (X + \overline{Y})$

Show the stages of your working.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

(3)

Final answer _____

(1)

(Total 9 marks)

Q19.

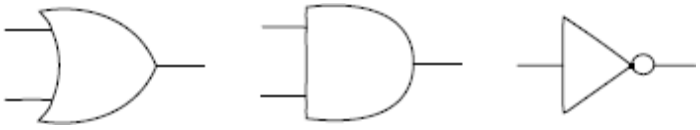
(a) Complete the truth tables for the following logic gates.

NAND Gate		
Input X	**Input Y**	**Output Q**
0	0	
0	1	
1	0	
1	1	

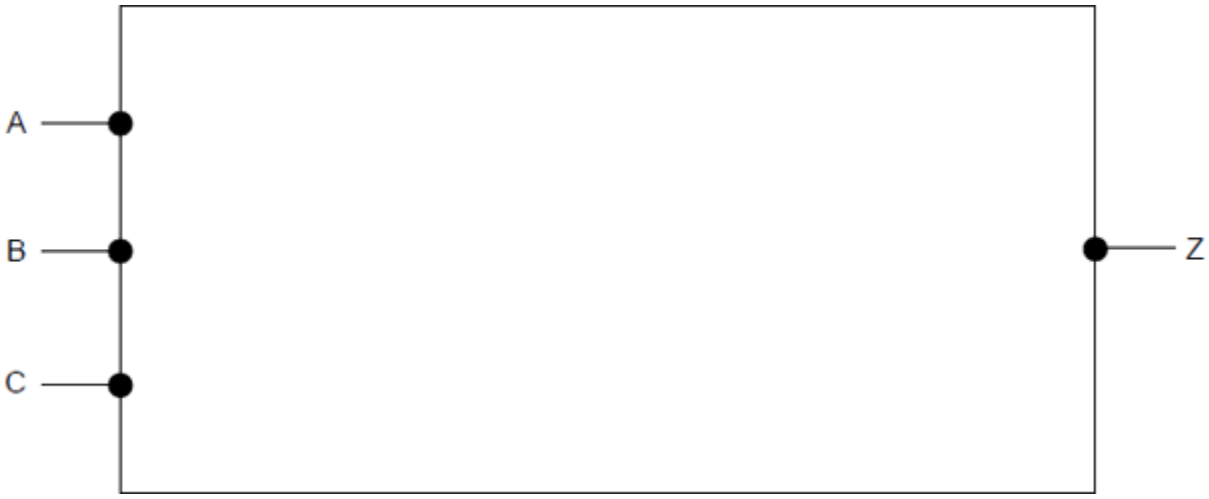
NOR Gate		
Input X	**Input Y**	**Output Q**
0	0	
0	1	
1	0	
1	1	

(2)

(b) Represent the Boolean equation Z = $\overline{A}$. $\overline{B}$ + C in the form of a logic circuit by drawing a diagram using the following symbols.



OR AND NOT



(3)

(c) Simplify the Boolean expression below.

$$\overline{(A \cdot B)} + \overline{(A \cdot \overline{B})}$$

Show each stage of your working.

(3)

Final answer _____

(1)

(Total 9 marks)

Q20.

Write the following Boolean expressions in their simplest forms.

(a) $\overline{(\overline{A} \cdot \overline{B})}$

_____

(1)

(b) $B + B \cdot \overline{C}$

_____

(1)

(c) $A \cdot B + A \cdot \overline{B}$

_____

(1)

(d) A . (B+1)

_____

(1)

(Total 4 marks)

Q21.

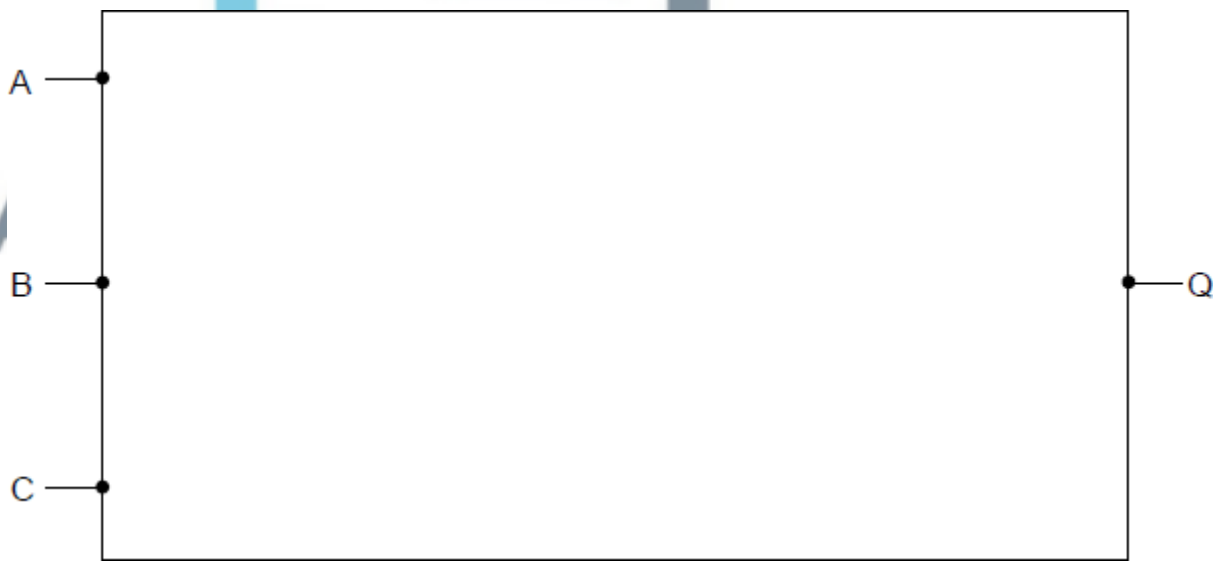
(a) Complete the truth tables for the following logic gates.

OR Gate		
Input A	**Input B**	**Output Q**
0	0	
0	1	
1	0	
1	1	

XOR Gate		
Input A	**Input B**	**Output Q**
0	0	
0	1	
1	0	
1	1	

(2)

(b) Represent the Boolean equation $Q = A + B \cdot \overline{C}$ as a logic circuit by drawing a diagram of it.



(3)

(c) Simplify the Boolean expression:

$$B \cdot (A + \overline{B})$$

Show your working.

_____

(3)

(Total 8 marks)

Q22.

Simplify the Boolean expression:

$$\overline{A \cdot B} + A$$

Show your working.

(Total 3 marks)

Q23.

(a) Look at the truth table below.

Input A	Input B	Output Q
0	0	1
0	1	0
1	0	0
1	1	0

What logic gate does the table represent?

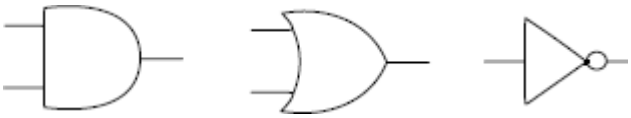
(1)

(b) An interior light in a two-door car is controlled by two switches that the driver can turn on or off and two sensors, one per door.

The switches are named A and B.
The door sensors are named C and D.
The interior light is named L.
If a door is open the output of its sensor is on.
If a door is closed the output of its sensor is off.

- If both switches A and B are off then the light L is always off.
- If switch A is on the light L is always on.
- If switch B is on and switch A is off then:
  - the light L turns on if one or more of the car doors is opened
  - the light L turns off if both of the doors are closed.

The following symbols are used to represent logic gates:



AND OR NOT

(i) Using only AND, OR and NOT gates draw a logic circuit for this system in the box below. You may not need to use all three types of gate.

(3)

(ii) Write a Boolean expression to represent the logic of the interior light system.

_____

(1)

(c) Simplify the Boolean expression below, showing your working.

$$\overline{\overline{A} + \overline{B}} + B \cdot \overline{A}$$

_____

_____

_____

_____

_____

_____

_____

_____

_____

(3)

(Total 8 marks)

Q24.

(a) Complete the truth tables for the following logic gates.

OR gate

Input A	Input B	Output
0	0	
0	1	
1	0	
1	1	

AND gate

Input A	Input B	Output
0	0	
0	1	
1	0	
1	1	

(2)

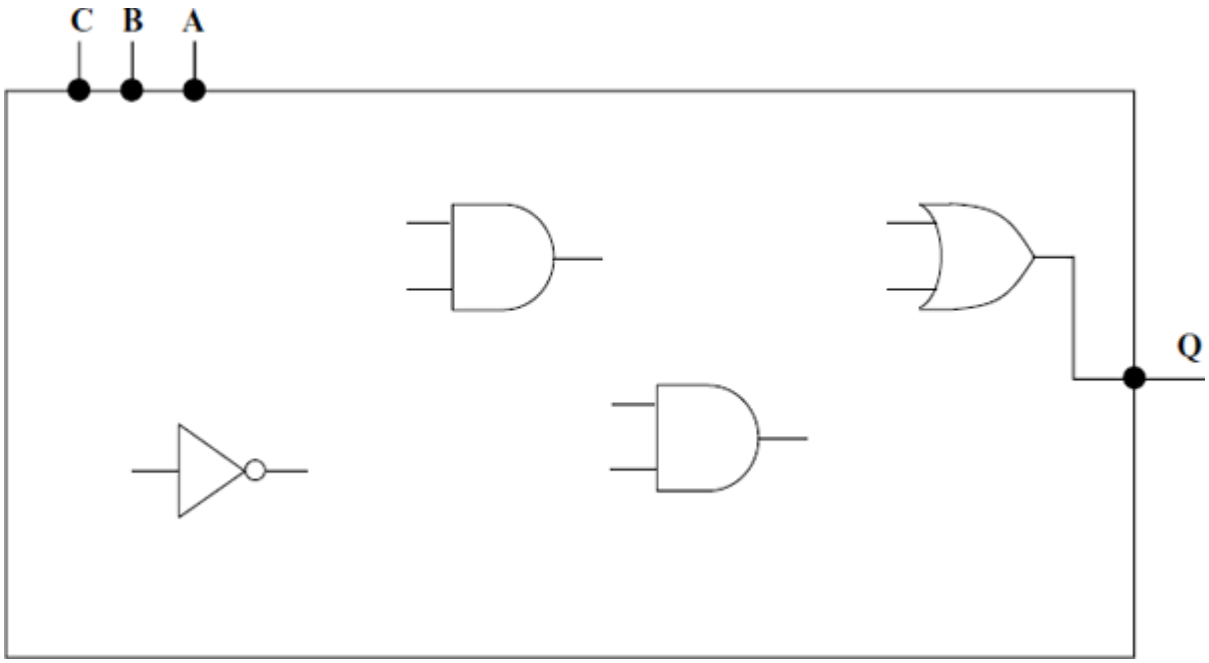
(b) (i) A single output Q is produced from three inputs A, B and C. Output Q is required to be 1 only if inputs A and B are 1, or input C is 1 and input B is 0.

Express this as a Boolean equation.

Q = _____

(2)

(ii) Represent this Boolean equation diagrammatically by completing the logic gate diagram below.



(4)
(Total 8 marks)