

Monday 15 May 2023 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

- 1 The roots of the equation $4x^4 - 2x^3 - 3x + 2 = 0$ are α , β , γ and δ . By using a suitable substitution, find a quartic equation whose roots are $\alpha + 2$, $\beta + 2$, $\gamma + 2$ and $\delta + 2$ giving your answer in the form $at^4 + bt^3 + ct^2 + dt + e = 0$, where a , b , c , d , and e are integers. [5]

- 2 The lines L_1 and L_2 have the following equations.

$$L_1 : \mathbf{r} = \begin{pmatrix} -5 \\ 6 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ -2 \end{pmatrix}$$

$$L_2 : \mathbf{r} = \begin{pmatrix} 24 \\ 1 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$$

- (a) Show that L_1 and L_2 intersect, giving the position vector of the point of intersection. [5]
- (b) Find the equation of the line which intersects L_1 and L_2 and is perpendicular to both. Give your answer in cartesian form. [3]

- 3 **In this question you must show detailed reasoning.**

In this question the principal argument of a complex number lies in the interval $[0, 2\pi)$.

Complex numbers z_1 and z_2 are defined by $z_1 = 3 + 4i$ and $z_2 = -5 + 12i$.

- (a) Determine $z_1 z_2$, giving your answer in the form $a + bi$. [2]
- (b) Express z_2 in modulus-argument form. [3]
- (c) Verify, by direct calculation, that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$. [3]

- 4 The vector \mathbf{p} , all of whose components are positive, is given by $\mathbf{p} = \begin{pmatrix} a^2 \\ a - 5 \\ 26 \end{pmatrix}$ where a is a constant.

You are given that \mathbf{p} is perpendicular to the vector $\begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$.

Determine the value of a . [4]

5 In this question you must show detailed reasoning.

The roots of the equation $5x^2 - 3x + 12 = 0$ are α and β .

By considering the symmetric functions of the roots, $\alpha + \beta$ and $\alpha\beta$, determine the exact value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. [4]

6 Prove by induction that $4 \times 8^n + 66$ is divisible by 14 for all integers $n \geq 0$. [6]

7 In this question you must show detailed reasoning.

Matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} a & -6 & a-3 \\ a+9 & a & 4 \\ 0 & -13 & a-1 \end{pmatrix}$ where a is a constant.

Find all possible values of a for which $\det \mathbf{A}$ has the same value as it has when $a = 2$. [6]

8 (a) Solve the equation $\omega + 2 + 7i = 3\omega^* - i$. [4]

(b) Prove algebraically that, for non-zero z , $z = -z^*$ if and only if z is purely imaginary. [2]

(c) The complex numbers z and z^* are represented on an Argand diagram by the points A and B respectively.

(i) State, for any z , the single transformation which transforms A to B . [1]

(ii) Use a geometric argument to prove that $z = z^*$ if and only if z is purely real. [2]

Turn over for question 9

9 Matrix \mathbf{R} is given by $\mathbf{R} = \begin{pmatrix} a & 0 & -b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix}$ where a and b are constants.

(a) Find \mathbf{R}^2 in terms of a and b . [2]

The constants a and b are given by $a = \frac{\sqrt{2}}{4}(\sqrt{3} + 1)$ and $b = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)$.

(b) By determining exact expressions for ab and $a^2 - b^2$ and using the result from part (a),

show that $\mathbf{R}^2 = k \begin{pmatrix} \sqrt{3} & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & \sqrt{3} \end{pmatrix}$ where k is a real number whose value is to be determined.

[2]

(c) Find \mathbf{R}^6 , \mathbf{R}^{12} and \mathbf{R}^{24} . [3]

(d) Describe fully the transformation represented by \mathbf{R} . [3]

END OF QUESTION PAPER

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