OCR
Oxford Cambridge and RSA

Monday 15 May 2023 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

Time allowed: 1 hour 15 minutes

9974108416

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 The roots of the equation $4x^4 - 2x^3 - 3x + 2 = 0$ are α, β, γ and δ. By using a suitable substitution, find a quartic equation whose roots are $\alpha + 2$, $\beta + 2$, $\gamma + 2$ and $\delta + 2$ giving your answer in the form $at^4 + bt^3 + ct^2 + dt + e = 0$, where a, b, c, d, and e are integers. **[5]**

2 The lines L_1 and L_2 have the following equations.

$$L_1 : \mathbf{r} = \begin{pmatrix} -5 \\ 6 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ -2 \end{pmatrix}$$

$$L_2 : \mathbf{r} = \begin{pmatrix} 24 \\ 1 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$$

(a) Show that L_1 and L_2 intersect, giving the position vector of the point of intersection. **[5]**

(b) Find the equation of the line which intersects L_1 and L_2 and is perpendicular to both. Give your answer in cartesian form. **[3]**

3 **In this question you must show detailed reasoning.**

In this question the principal argument of a complex number lies in the interval $[0, 2\pi)$.

Complex numbers z_1 and z_2 are defined by $z_1 = 3 + 4\mathrm{i}$ and $z_2 = -5 + 12\mathrm{i}$.

(a) Determine $z_1 z_2$, giving your answer in the form $a + b\mathrm{i}$. **[2]**

(b) Express z_2 in modulus-argument form. **[3]**

(c) Verify, by direct calculation, that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$. **[3]**

4 The vector $\mathbf{p}$, all of whose components are positive, is given by $\mathbf{p} = \begin{pmatrix} a^2 \\ a-5 \\ 26 \end{pmatrix}$ where a is a constant.

You are given that $\mathbf{p}$ is perpendicular to the vector $\begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$.

Determine the value of a. **[4]**

5 In this question you must show detailed reasoning.

The roots of the equation $5x^2 - 3x + 12 = 0$ are α and β.

By considering the symmetric functions of the roots, $\alpha + \beta$ and $\alpha\beta$, determine the exact value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. **[4]**

6 Prove by induction that $4 \times 8^n + 66$ is divisible by 14 for all integers $n \geqslant 0$. **[6]**

7 In this question you must show detailed reasoning.

Matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & -6 & a-3 \\ a+9 & a & 4 \\ 0 & -13 & a-1 \end{pmatrix}$ where a is a constant.

Find all possible values of a for which det **A** has the same value as it has when $a = 2$. **[6]**

8 (a) Solve the equation $\omega + 2 + 7\mathrm{i} = 3\omega^* - \mathrm{i}$. **[4]**

(b) Prove algebraically that, for non-zero z, $z = -z^*$ if and only if z is purely imaginary. **[2]**

(c) The complex numbers z and z^* are represented on an Argand diagram by the points A and B respectively.

(i) State, for any z, the single transformation which transforms A to B. **[1]**

(ii) Use a geometric argument to prove that $z = z^*$ if and only if z is purely real. **[2]**

Turn over for question 9

9 Matrix $\mathbf{R}$ is given by $\mathbf{R} = \begin{pmatrix} a & 0 & -b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix}$ where a and b are constants.

(a) Find $\mathbf{R}^2$ in terms of a and b. **[2]**

The constants a and b are given by $a = \frac{\sqrt{2}}{4}(\sqrt{3}+1)$ and $b = \frac{\sqrt{2}}{4}(\sqrt{3}-1)$.

(b) By determining exact expressions for ab and $a^2 - b^2$ and using the result from part **(a)**,

show that $\mathbf{R}^2 = k\begin{pmatrix} \sqrt{3} & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & \sqrt{3} \end{pmatrix}$ where k is a real number whose value is to be determined.

[2]

(c) Find $\mathbf{R}^6$, $\mathbf{R}^{12}$ and $\mathbf{R}^{24}$. **[3]**

(d) Describe fully the transformation represented by $\mathbf{R}$. **[3]**

END OF QUESTION PAPER