

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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## Pearson Edexcel International GCSE

Time 2 hours

Paper  
reference

**4MA1/2HR**

### Mathematics A

**PAPER: 2HR**

**Higher Tier**



**You must have:** Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.  
Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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International GCSE Mathematics

Formulae sheet – Higher Tier

**Arithmetic series**

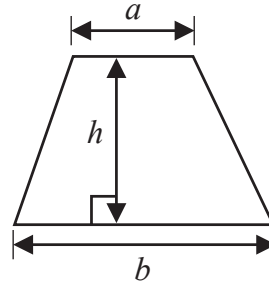
Sum to  $n$  terms,  $S_n = \frac{n}{2} [2a + (n - 1)d]$

**Area of trapezium** =  $\frac{1}{2}(a + b)h$

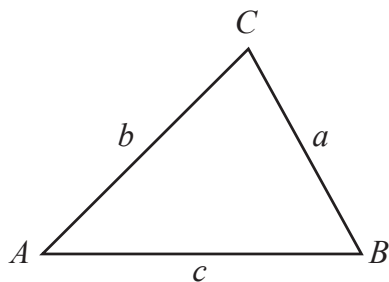
**The quadratic equation**

The solutions of  $ax^2 + bx + c = 0$  where  $a \neq 0$  are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



**Trigonometry**



**In any triangle ABC**

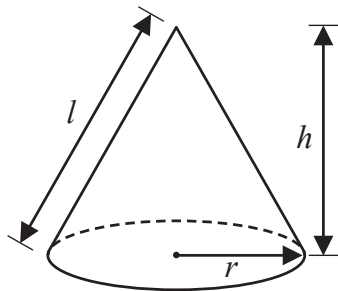
**Sine Rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine Rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

**Area of triangle** =  $\frac{1}{2}ab \sin C$

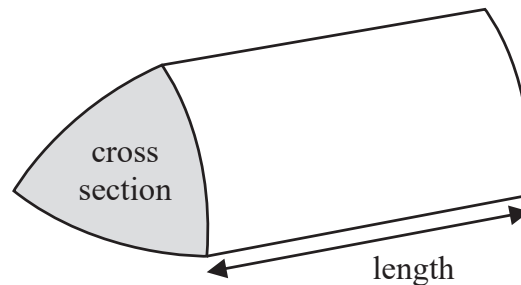
**Volume of cone** =  $\frac{1}{3}\pi r^2 h$

**Curved surface area of cone** =  $\pi r l$



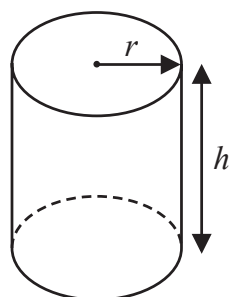
**Volume of prism**

= area of cross section  $\times$  length



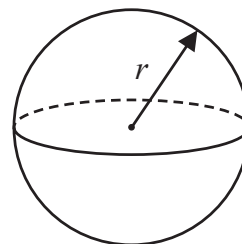
**Volume of cylinder** =  $\pi r^2 h$

**Curved surface area of cylinder** =  $2\pi r h$



**Volume of sphere** =  $\frac{4}{3}\pi r^3$

**Surface area of sphere** =  $4\pi r^2$



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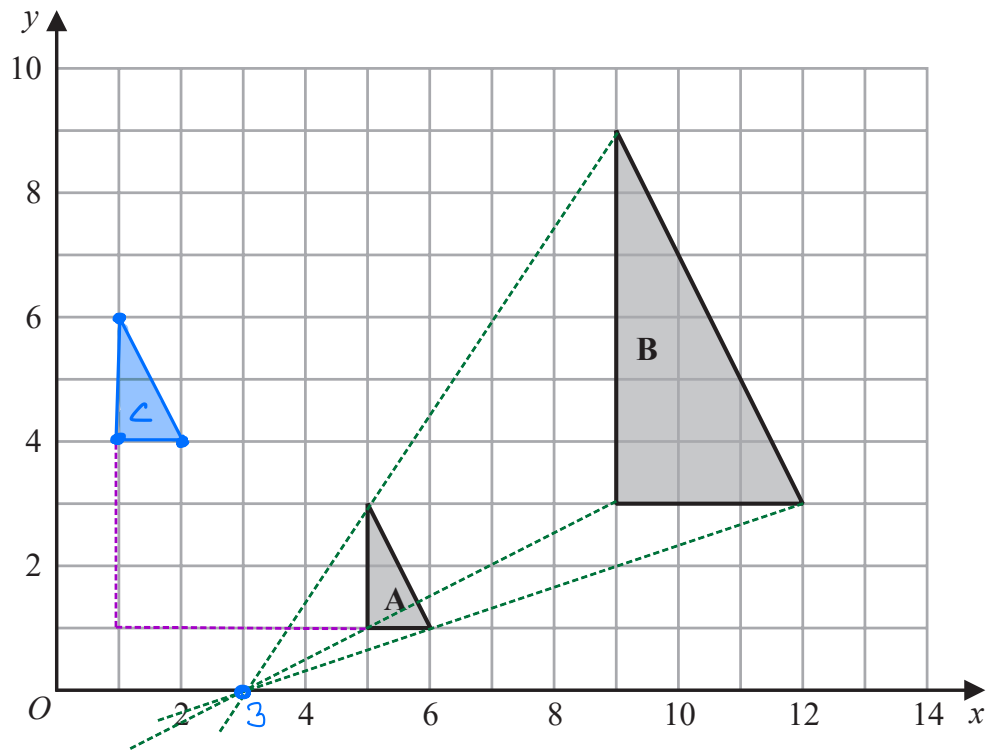


Answer ALL TWENTY FIVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1



- (a) Describe fully the single transformation that maps triangle **A** onto triangle **B**

enlargement,  $ef = 3$ , centre  $(3, 0)$

(3)

- (b) On the grid above, translate triangle A by the vector  $\begin{pmatrix} -4 \\ +3 \end{pmatrix}$

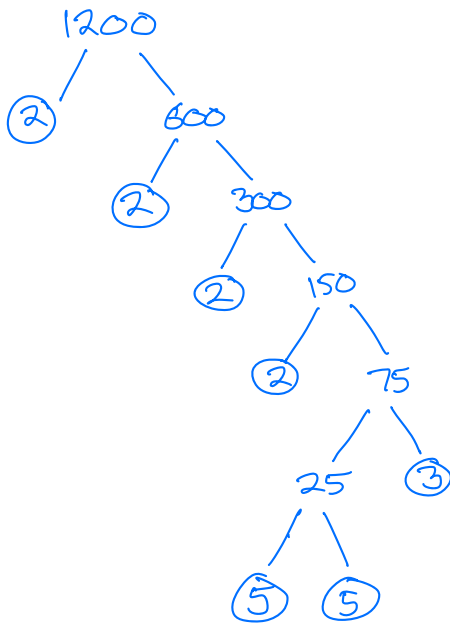
Label your triangle C

(1)

(Total for Question 1 is 4 marks)



- 2 Write 1200 as a product of powers of its prime factors.  
Show your working clearly.



$$1200 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5$$
$$= 2^4 \times 3 \times 5^2$$

$$2^4 \times 3 \times 5^2$$

(Total for Question 2 is 3 marks)

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3 Alberto, Bill, Candela and Diana are four friends.

Here is some information about the height of each of these friends.

Alberto's height is 158 cm.

Bill's height is 175 cm.

Candela's height is greater than Diana's height.

The median height of these four friends is 160 cm.

The **range** of the heights of these four friends is **21 cm**.

Work out Candela's height and Diana's height.

$175 - 158 = 17 < \text{range}$  so C and cannot both be between A + B  
 median = 160  $\rightarrow$  must be between A and x  
 (between x + B would mean  $x = 145 < 158 \times$ )  
 $\Rightarrow x = 162$ ; must be C

|   |     |     |     |
|---|-----|-----|-----|
| D | A   | C   | B   |
| y | 158 | 162 | 175 |

range = 17  
 $\Rightarrow y = 175 - 21$   
 $= 154$

$\rightarrow$  C: 162cm

D: 154cm

Candela ..... 162 ..... cm

Diana ..... 154 ..... cm

(Total for Question 3 is 3 marks)



4  $\mathcal{E} = \{9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A = \{\text{multiples of 3}\} = \{9, 12, 15, 18\}$

$B = \{\text{odd numbers}\} = \{9, 11, 13, 15, 17, 19\}$

(a) List the members of the set

(i)  $A \cap B$

↑ intersection: And  
in both A and B

9, 15

(1)

(ii)  $A \cup B$

↑ union  
in A or B or both  
but don't write elements twice

9, 11, 12, 13, 15, 17, 18, 19

(1)

(b) Is it true that  $24 \in A$ ?

Tick one of the boxes below.

Yes

No



Give a reason for your answer.

24 is not in the universal set  $\mathcal{E}$

(1)

Set C has 4 members such that  $C \cap B' = \{10, 18\}$

(c) List the members of one possible set C

10 and 18 are in C if in  $C \cap B'$

→ remaining elements of C must be in B (or would be in  $C \cap B'$ )

any 2 of: 9, 11, 13, 15, 17, 19

eg 9, 10, 11, 18

(2)

(Total for Question 4 is 5 marks)



- 5 The diagram shows a shape made from a square  $ABCD$  and 4 identical semicircles.

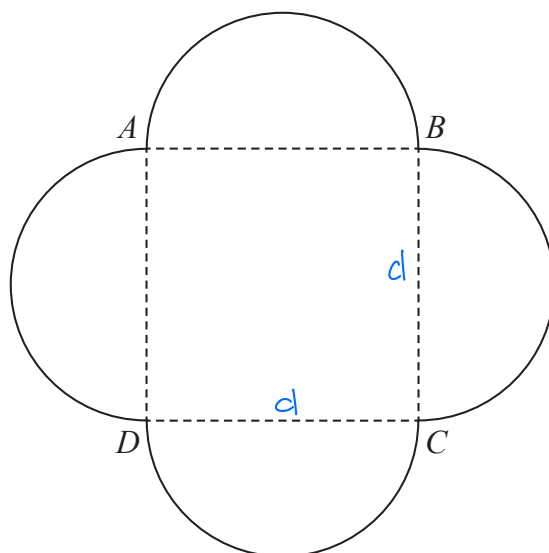


Diagram NOT  
accurately drawn

As shown in the diagram, the semicircles have  $AB$ ,  $BC$ ,  $CD$  and  $DA$  as diameters.

The area of the square is  $36 \text{ cm}^2$

Calculate the total area of the shape.

Give your answer correct to one decimal place.

$$\text{Area of square} = d^2 = 36$$

$$\Rightarrow d = \sqrt{36} = 6 \text{ cm}$$

→ diameter of semicircles

$$r = \frac{6}{2} = 3 \text{ cm}$$

4 semicircles = 2 whole circles

$$\rightarrow A = 2 \times \pi \times 3^2$$

$$= 56.5486\dots$$

$$A(\text{circle}) = \pi r^2$$

$$\text{Total area} = 56.5486\dots + 36$$

$$\approx 92.5 \text{ cm}^2$$

.....92.5.....  $\text{cm}^2$

(Total for Question 5 is 4 marks)



6 (a) Solve  $p = \frac{3p - 5}{10}$

Show clear algebraic working.

$$\begin{aligned} 10p &= 3p - 5 \\ -3p & \quad -3p \\ 7p &= -5 \\ \div 7 & \quad \div 7 \\ p &= \frac{-5}{7} \end{aligned}$$

$$p = \frac{-5}{7} \dots\dots\dots (3)$$

(b) Simplify  $a^0$  where  $a > 0$

$$\text{anything}^0 = 1$$

$$\dots\dots\dots \frac{1}{\dots\dots\dots} \dots\dots\dots (1)$$

(c) Simplify fully  $\frac{3xy^3}{6x^2y}$

$$\frac{\cancel{3} \times \cancel{x} \times y \times y \times y}{2 \times \cancel{3} \times \cancel{x} \times x \times y} = \frac{y^2}{2x}$$

$$\dots\dots\dots \frac{y^2}{2x} \dots\dots\dots (2)$$

(d) Factorise fully  $10c^3d^2 + 15cd^4$

$$\begin{aligned} &= \underline{2 \times 5} \times \underline{c \times c \times c} \times \underline{d \times d} + \underline{3 \times 5} \times \underline{c} \times \underline{d \times d \times d \times d} \\ &= \underline{5cd^2} (\underline{2c^2} + \underline{3d^2}) \end{aligned}$$

$$\dots\dots\dots 5cd^2(2c^2 + 3d^2) \dots\dots\dots (2)$$

(Total for Question 6 is 8 marks)





$$7 \quad \frac{2^k}{4^n} = 2^x$$

Find an expression for  $x$  in terms of  $k$  and  $n$

Need to express everything as a power of 2

$$4 = 2^2: \quad \frac{2^k}{4^n} = \frac{2^k}{(2^2)^n}$$

$$= \frac{2^k}{2^{2n}}$$

$$= 2^{k-2n}$$

$$(a^m)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\Rightarrow 2^{k-2n} = 2^x$$

$$\Rightarrow k-2n = x$$

$$x = k - 2n$$

(Total for Question 7 is 2 marks)

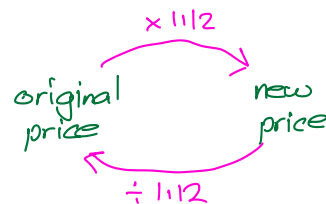
8 A cinema increased the cost of an adult ticket by 12%

$$100 + 12 = 112\% = 1.12$$

After the increase, the cost of an adult ticket was £18.20

Work out the cost of an adult ticket before the increase.

↑  
reversing change  
 $\Rightarrow \div$  decimal multiplier



$$\frac{18.20}{1.12} = \pounds 16.25$$

$$\pounds 16.25$$

(Total for Question 8 is 3 marks)



- 9 The table gives information about the population, correct to 2 significant figures, of each of five cities in 2018

| City      | Population (2018) |
|-----------|-------------------|
| Ahmedabad | $7.7 \times 10^6$ |
| Barcelona | $5.5 \times 10^6$ |
| Chicago   | $8.8 \times 10^6$ |
| Lagos     | $1.3 \times 10^7$ |
| Tokyo     | $3.7 \times 10^7$ |

- (a) Write  $8.8 \times 10^6$  as an ordinary number.

$$8.8 \rightarrow 6 \text{ place right}$$

$$8\ 800\ 000$$

$$8\ 800\ 000$$

(1)

- (b) Which of these cities had the least population in 2018?

first look for smallest power of 10:  $10^6$

then smallest number with this power

$$5.5 \times 10^6 : \text{Barcelona}$$

$$\text{Barcelona}$$

(1)

- (c) Work out the difference between the population of Tokyo and the population of Ahmedabad in 2018

Give your answer in standard form correct to 2 significant figures.

$$(3.7 \times 10^7) - (7.7 \times 10^6)$$

use  $\boxed{\times 10^x}$  button on calculator

$$= 29\ 300\ 000$$

$$= 2.93 \times 10^7$$

$$\approx 2.9 \times 10^7 \quad (2\text{sf})$$

$$2.9 \times 10^7$$

(2)

(Total for Question 9 is 4 marks)



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10 The diagram shows triangle  $ABP$  inside the regular hexagon  $ABCDEF$

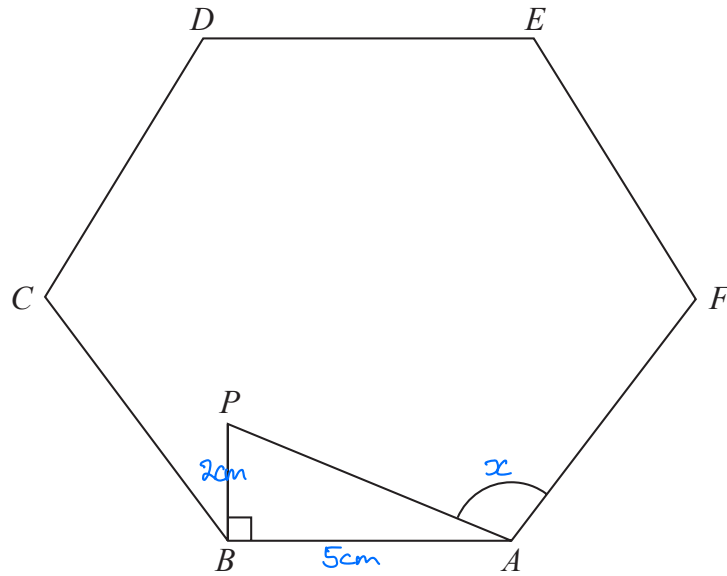


Diagram NOT accurately drawn

$$AB = 5 \text{ cm}$$

$$BP = 2 \text{ cm}$$

$$\text{Angle } ABP = 90^\circ$$

Work out the size of angle  $PAF$

Give your answer correct to 3 significant figures.

$ABCDEF$  is regular hexagon

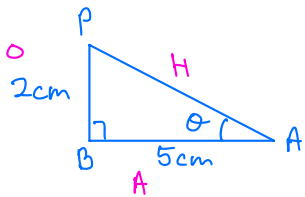
$$\Rightarrow e = \frac{360}{6} = 60^\circ$$

exterior angle

interior angle

$$i = 180 - 60 = 120^\circ$$

$$\Rightarrow \angle BAF = 120^\circ$$



$$\text{O/A} \rightarrow \text{"TOA"} \quad \tan \theta = \frac{\text{O}}{\text{A}}$$

$$\tan \theta = \frac{2}{5}$$

$$\theta = \tan^{-1}\left(\frac{2}{5}\right)$$

$$\approx 21.8^\circ$$

$$\angle PAF = 120 - 21.8$$

$$\approx 98.2^\circ$$

..... 98.2 °

(Total for Question 10 is 5 marks)

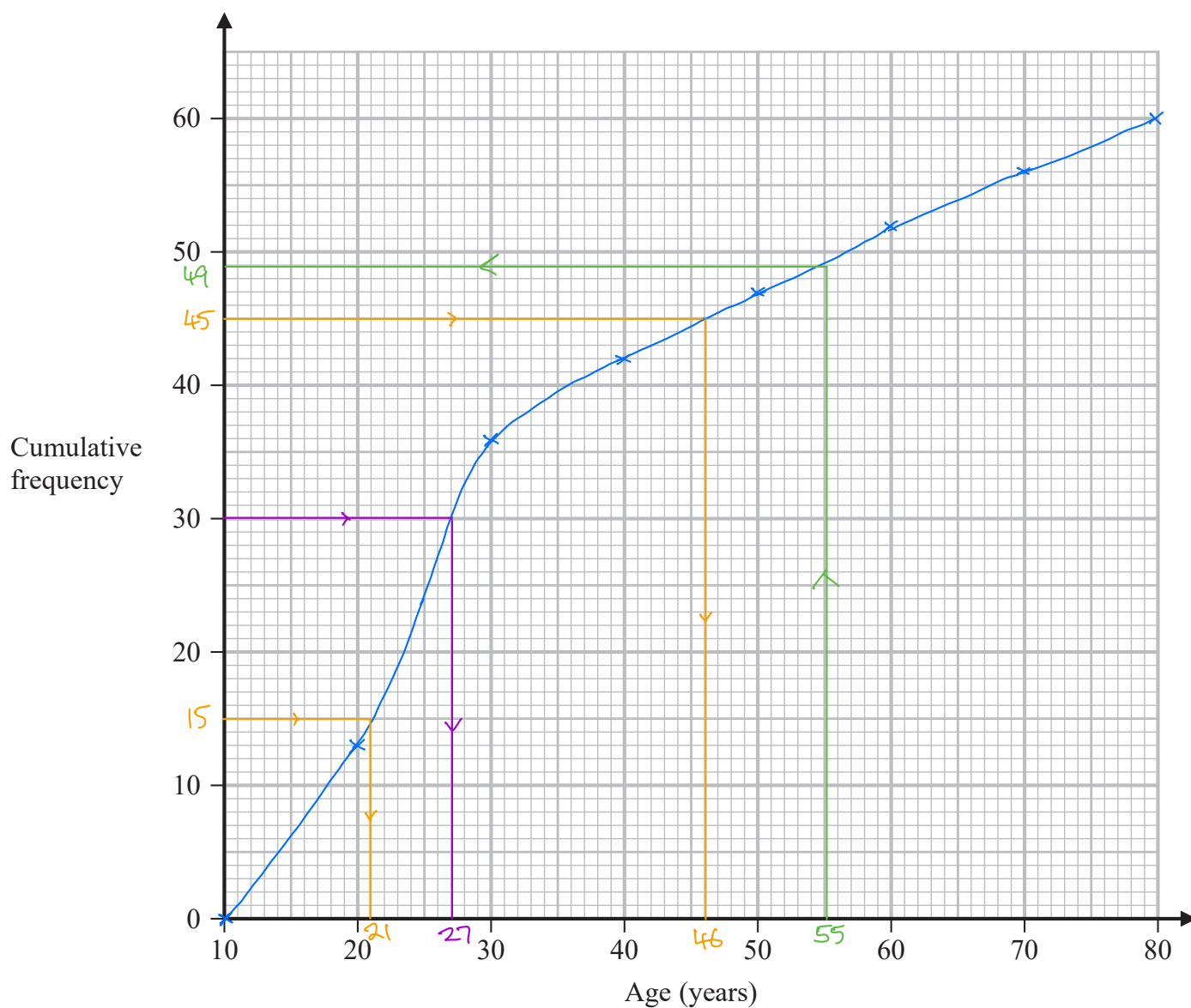


P 6 8 7 9 1 A 0 1 1 2 8

- 11 The cumulative frequency table shows information about the ages of 60 people who went to a gym on Saturday.

| Age ( $a$ years) | Cumulative frequency |
|------------------|----------------------|
| $10 < a \leq 20$ | 13                   |
| $10 < a \leq 30$ | 36                   |
| $10 < a \leq 40$ | 42                   |
| $10 < a \leq 50$ | 47                   |
| $10 < a \leq 60$ | 52                   |
| $10 < a \leq 70$ | 56                   |
| $10 < a \leq 80$ | 60                   |

- (a) On the grid, draw a cumulative frequency graph for the information in the table.



(2)



Question 11 continued

(b) Use your graph to find an estimate for the median of the ages of these people.

$$cf = \frac{60}{2} = 30 : Q_2 = 27$$

(26-28)

..... 27 ..... years  
(1)

(c) Use your graph to find an estimate for the interquartile range of the ages of these people.

$$cf = \frac{60}{4} = 15 : Q_1 = 21$$

$$cf = \frac{3}{4} \times 60 = 45 : Q_3 = 46$$

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 46 - 21 = 25 \end{aligned}$$

(24-28)

..... 25 ..... years  
(2)

(d) Use your graph to find an estimate for the number of these people who are older than 55 years.

$$\text{age} = 55 : cf = 49 \Rightarrow 49 \text{ people} < 55$$

$$\Rightarrow 60 - 49 = 11 \text{ people} > 55$$

..... 11 .....  
(2)

(Total for Question 11 is 7 marks)

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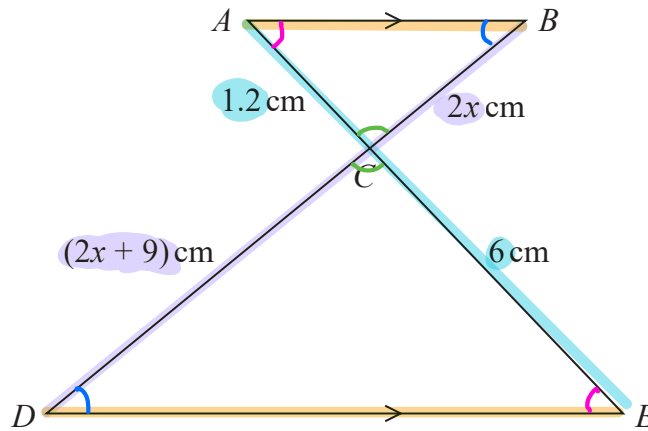


Diagram NOT  
accurately drawn

$ACE$  and  $BCD$  are straight lines.

$AB$  is parallel to  $DE$

Work out the value of  $x$

Triangles  $ABC$  and  $CDE$  are similar (corresponding sides highlighted)

$$\frac{AC}{CE} = \frac{BC}{CD} \Rightarrow \frac{1.2}{6} = \frac{2x}{2x+9}$$

$$\Rightarrow 6 \times 2x = 1.2(2x+9)$$

$$12x = 2.4x + 10.8 \quad (\times 5)$$

$$60x = 12x + 54 \quad (\div 6)$$

$$10x = 2x + 9$$

$$8x = 9$$

$$x = \frac{9}{8}$$

$$x = \frac{9}{8}$$

(Total for Question 12 is 3 marks)



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13 The diagram shows a sector  $AOB$  of a circle with centre  $O$

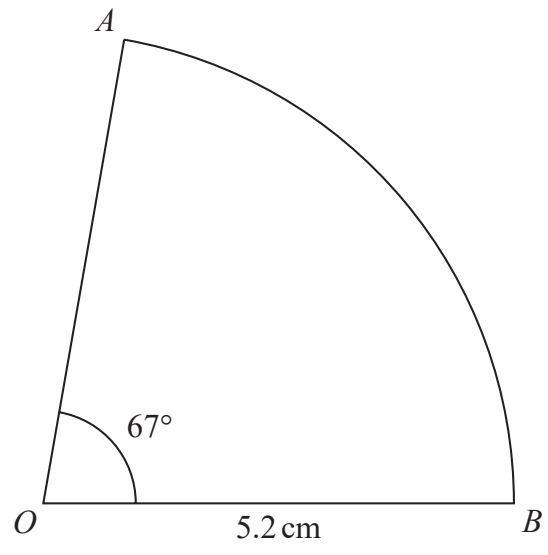


Diagram NOT accurately drawn

Angle  $AOB = 67^\circ$   
 $OA = OB = 5.2 \text{ cm}$

Calculate the perimeter of the sector.  
 Give your answer correct to 3 significant figures.

$$\text{arc length} = \frac{67}{360} \times 2 \times \pi \times 5.2 \quad L = \frac{\theta}{360} \times 2\pi r$$

$$= 6.0807\dots$$

$$p = 6.0807\dots + 2 \times 5.2$$

$$\approx 16.5 \text{ cm}$$

..... 16.5 ..... cm

(Total for Question 13 is 3 marks)



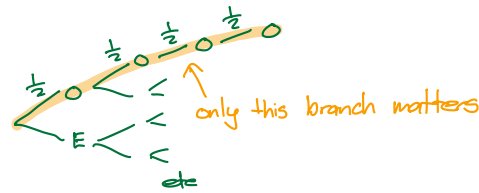
14 Ciara throws **four** fair six-sided dice.

The faces of each dice are labelled with the numbers 1, 2, 3, 4, 5, 6

Work out the probability that at least one of the dice lands on an even number.

for 1 die :  $P(\text{even}) = P(\text{odd}) = \frac{1}{2}$

$$\begin{aligned} P(\text{at least 1 even}) &= 1 - P(\text{all odd}) \\ &= 1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{15}{16} \end{aligned}$$



.....  $\frac{15}{16}$  .....

(Total for Question 14 is 3 marks)

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15 The diagram shows a kite  $ABCD$

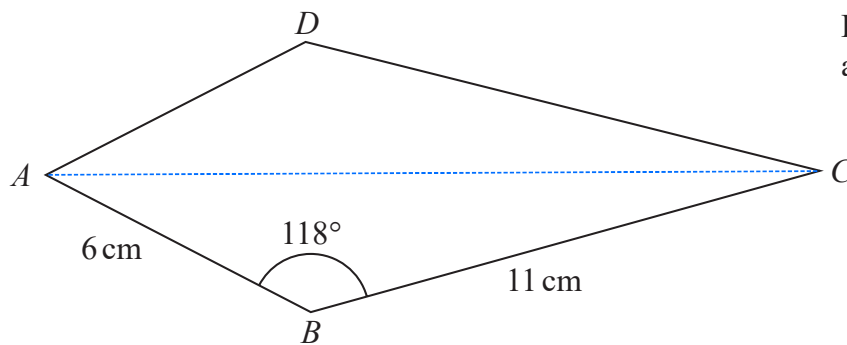


Diagram **NOT** accurately drawn

$$AB = 6\text{ cm}$$

$$BC = 11\text{ cm}$$

$$\text{Angle } ABC = 118^\circ$$

Calculate the area of the kite.

Give your answer correct to 3 significant figures.

$$\begin{aligned} \text{Area of kite} &= 2 \times \text{Area of } \triangle ABC \\ &= 2 \times \frac{1}{2} \times 6 \times 11 \times \sin 118^\circ \\ &\approx 58.3\text{ cm}^2 \end{aligned}$$

$$\text{Sine area rule: Area} = \frac{1}{2} ab \sin C$$

$$\dots\dots\dots 58.3 \dots\dots \text{ cm}^2$$

(Total for Question 15 is 3 marks)

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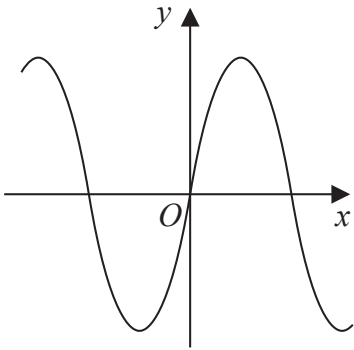
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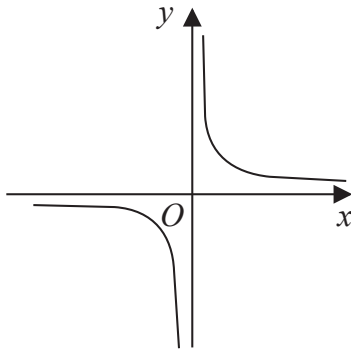


16 Here are nine graphs.

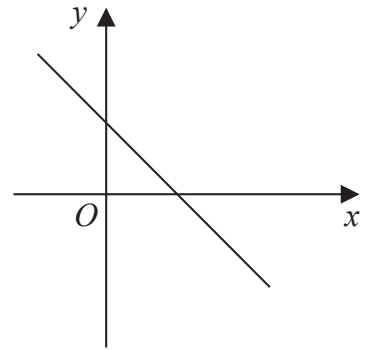
**Graph A**



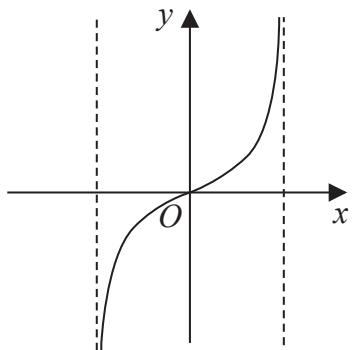
**Graph B**



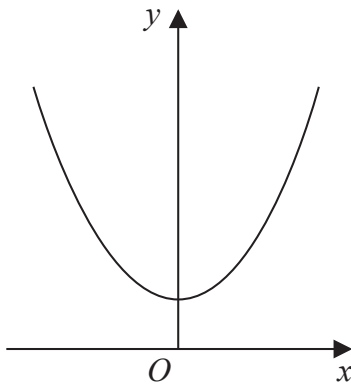
**Graph C**



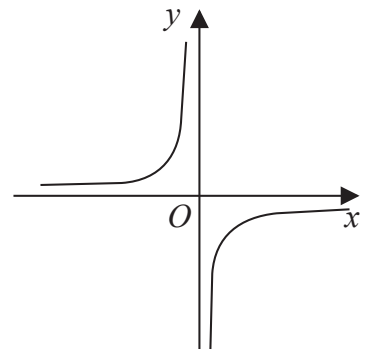
**Graph D**



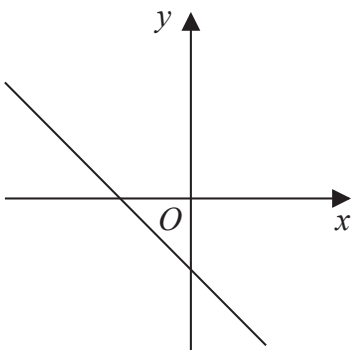
**Graph E**



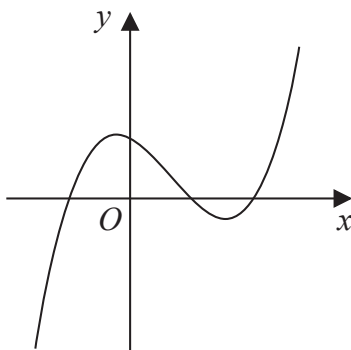
**Graph F**



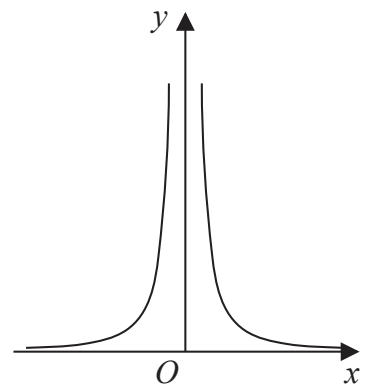
**Graph G**



**Graph H**



**Graph I**



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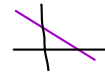
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Complete the table below with the letter of the graph that could represent each given equation. Write each answer on the dotted line.

| Equation                    | Graph         |
|-----------------------------|---------------|
| $y = -2x + 3$               | ..... C ..... |
| $y = -\frac{1}{x}$          | ..... F ..... |
| $y = \tan x^\circ$          | ..... D ..... |
| $y = (x + 1)(x - 1)(x - 2)$ | ..... H ..... |

straight line:  
 $m < 0$   
 $c > 3$



reciprocal "1-"  
 $\Rightarrow$



$\tan x$



cubic  $x^3 > 0$ :  
 $x$ -intercepts  $-1, 1, 2$



(Total for Question 16 is 3 marks)

17 Use algebra to show that  $0.3\dot{4}\dot{5} = \frac{19}{55}$

2 recurring digits:  $\times 10^2$  ( $\times 100$ )

$$x = 0.3\dot{4}\dot{5} = 0.3454545\dots$$

$$100x = 34.5\dot{4}\dot{5}454\dots$$

$$- x = 0.3\dot{4}\dot{5}454\dots$$

$$99x = 34.2$$

$$x = \frac{34.2}{99}$$

$$= \frac{342}{990}$$

$$= \frac{19}{55} \quad \checkmark \checkmark$$

(Total for Question 17 is 2 marks)



18 Kaidan and Sonja went on two different car journeys.

For Kaidan's journey

distance = 80 km correct to the nearest 5 km      77.5 - 82.5 km  
 time = 2.7 hours correct to 1 decimal place      2.65 - 2.75 h

For Sonja's journey

distance = 33 km correct to 2 significant figures      32.5 - 33.5 km  
 time = 1 hour correct to the nearest 0.1 hour      0.95 - 1.05 h

Kaidan says,

"My average speed could have been greater than Sonja's average speed."

By considering bounds, show that Kaidan is correct.

Show your working clearly.

$$v = \frac{d}{t}$$

$$K: UB(v) = \frac{UB(d)}{LB(t)}$$

$$= \frac{82.5}{2.65}$$

$$= 31.132\dots$$

$$S: LB(v) = \frac{LB(d)}{UB(t)}$$

$$= \frac{32.5}{1.05}$$

$$= 30.952\dots$$

upper bound of average speed for K > lower bound of average speed for S

$\Rightarrow$  Kaidan is correct

(Total for Question 18 is 4 marks)



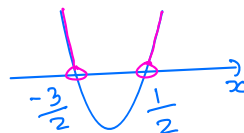
19  $f(x) = x^2 - 4$

$$g(x) = 2x + 1$$

Solve  $fg(x) > 0$

Show clear algebraic working.

$$\begin{aligned} fg(x) &= f[g(x)] \\ &= (2x+1)^2 - 4 > 0 \end{aligned}$$



$$(2x+1)^2 - 4 = 0$$

$$(2x+1)^2 = 4$$

$$2x+1 = \pm 2$$

$$2x = -3 \quad \text{or} \quad 2x = 1$$

$$x = -\frac{3}{2} \quad \quad \quad x = \frac{1}{2}$$

$$\rightarrow x < -\frac{3}{2} \quad \text{or} \quad x > \frac{1}{2}$$

$$x < -\frac{3}{2}, x > \frac{1}{2}$$

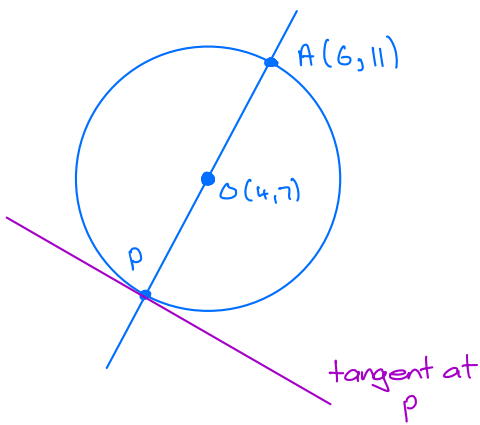
(Total for Question 19 is 4 marks)



20 The centre  $O$  of a circle has coordinates  $(4, 7)$

The point  $A$ , on the circle, has coordinates  $(6, 11)$  and  $AOP$  is a diameter of the circle.

Find an equation of the tangent to the circle at the point  $P$



tangent at  $P$   $\perp$  line  $AOP$  (tangent  $\perp$  radius of circle)  
and goes through  $P$

$O$  is midpoint of  $AP$   $P(x, y)$

$$\Rightarrow \frac{x+6}{2} = 4 \quad \frac{y+11}{2} = 7$$

$$x = 2 \quad y = 3$$

$$P(2, 3)$$

$$m_{PA} = \frac{8}{4} = 2 \quad \Rightarrow m_{\perp} = -\frac{1}{2}$$

$$\text{tangent at } P: y = -\frac{1}{2}x + c$$

$$P(2, 3) \text{ on tangent: } 3 = -\frac{1}{2} \times 2 + c$$

$$c = 4$$

$$\rightarrow y = -\frac{1}{2}x + 4$$

$$y = -\frac{1}{2}x + 4$$

(Total for Question 20 is 4 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



21 Solve the simultaneous equations

$$\begin{aligned}x - 2y &= 3 & \textcircled{1} \\x^2 - y^2 + 2x &= 10 & \textcircled{2}\end{aligned}$$

Show clear algebraic working.

$$\textcircled{1} : x = 2y + 3$$

$$\begin{aligned}\text{in } \textcircled{2} : (2y+3)^2 - y^2 + 2(2y+3) &= 10 \\4y^2 + 12y + 9 - y^2 + 4y + 6 - 10 &= 0\end{aligned}$$

$$3y^2 + 16y + 5 = 0$$

$$(3y+1)(y+5) = 0$$

$$y = -\frac{1}{3}, -5$$

$$\text{in } \textcircled{1} : y = -\frac{1}{3} : x = 2 \times \left(-\frac{1}{3}\right) + 3 = \frac{7}{3}$$

$$y = -5 : x = 2 \times (-5) + 3 = -7$$

$$\begin{aligned}(2y+3)^2 &= (2y+3)(2y+3) \\&= 4y^2 + 6y + 6y + 9 \\&= 4y^2 + 12y + 9\end{aligned}$$

$$x = \frac{7}{3}, y = -\frac{1}{3}$$

$$\text{or } x = -7, y = -5$$

(Total for Question 21 is 5 marks)



22 The point  $A$  with coordinates  $(-3, 2)$  lies on the straight line with equation  $y = f(x)$

(a) Find the coordinates of the image of the point  $A$  on the straight line with equation

(i)  $y = f(x) - 3$

$$(x, y) \rightarrow (x, y - 3)$$

$$\left( \dots \overset{-3}{-3} \dots, \dots \overset{-1}{-1} \dots \right)$$

(1)

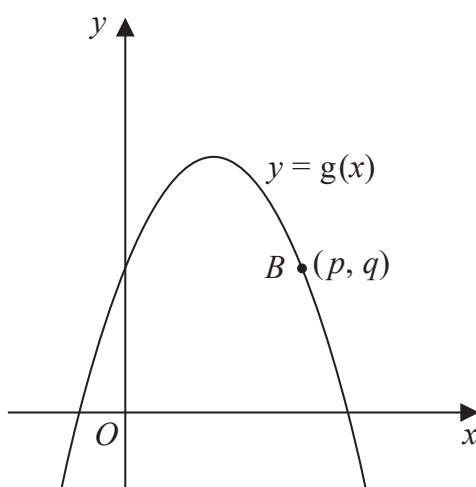
(ii)  $y = f\left(\frac{x}{2}\right)$

$$(x, y) \rightarrow (2x, y)$$

$$\left( \dots \overset{-6}{-6} \dots, \dots \overset{2}{2} \dots \right)$$

(1)

Here is a sketch of part of the curve with equation  $y = g(x)$



The point  $B$  with coordinates  $(p, q)$  lies on the curve.

(b) Find the coordinates of the image of the point  $B$  on the curve with equation

$$y = -g(x - c)$$

where  $c$  is a constant.

$$(x, y) \rightarrow (x + c, -y)$$

$$\Rightarrow (p, q) \rightarrow (p + c, -q)$$

$$\left( \dots \overset{p+c}{p+c} \dots, \dots \overset{-q}{-q} \dots \right)$$

(2)

(Total for Question 22 is 4 marks)





23 Express  $\left(\frac{20}{x^2-36} - \frac{2}{x-6}\right) \times \frac{1}{4-x}$  as a single fraction in its simplest form.

First factorise as much as possible:  $x^2 - 36 = (x+6)(x-6)$   
*difference of 2 squares*

$$\begin{aligned} & \left(\frac{20}{(x+6)(x-6)} - \frac{2}{x-6}\right) \times \frac{1}{4-x} && \text{Find lowest common denominator} \\ & = \left(\frac{20}{(x+6)(x-6)} - \frac{2(x+6)}{(x-6)(x+6)}\right) \times \frac{1}{4-x} && \text{for expression in ( )} \\ & = \frac{20 - (2x+12)}{(x+6)(x-6)} \times \frac{1}{4-x} \\ & = \frac{8 - 2x}{(x+6)(x-6)(4-x)} \\ & = \frac{2(4-x)}{(x+6)(x-6)(4-x)} \\ & = \frac{2}{(x+6)(x-6)} \end{aligned}$$

$$\frac{2}{(x+6)(x-6)}$$

(Total for Question 23 is 3 marks)



24 The diagram shows a frustum of a cone, and a sphere.

The frustum, shown shaded in the diagram, is made by removing the small cone from the large cone.

The small cone and the large cone are similar.

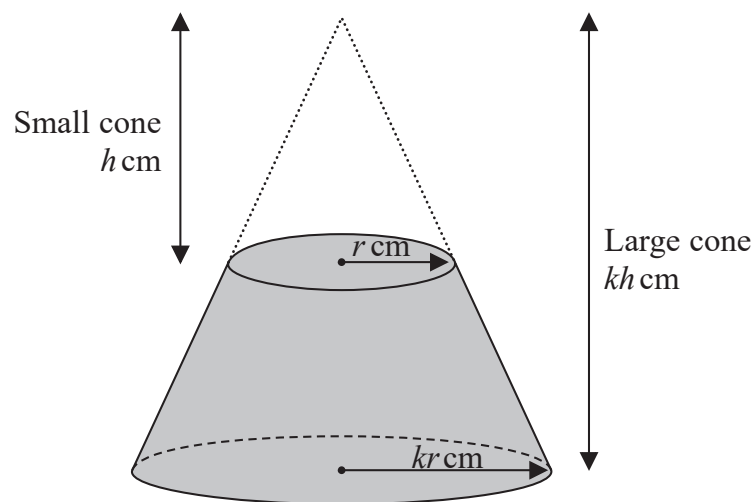
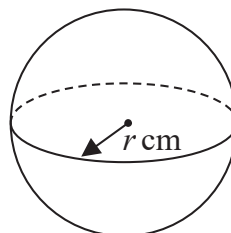


Diagram **NOT** accurately drawn



The height of the small cone is  $h$  cm and the radius of the base of the small cone is  $r$  cm. The height of the large cone is  $kh$  cm and the radius of the base of the large cone is  $kr$  cm. The radius of the sphere is  $r$  cm.

The sphere is divided into two hemispheres, each of radius  $r$  cm.

Solid **A** is formed by joining one of the hemispheres to the frustum.

The plane face of the hemisphere coincides with the upper plane face of the frustum, as shown in the diagram below.

Solid **B** is formed by joining the other hemisphere to the small cone that was removed from the large cone.

The plane face of the hemisphere coincides with the plane face of the base of the small cone, as shown in the diagram below.

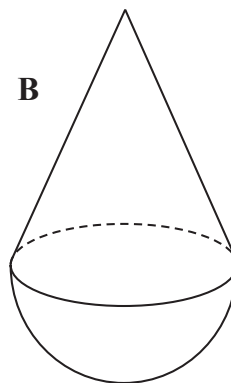
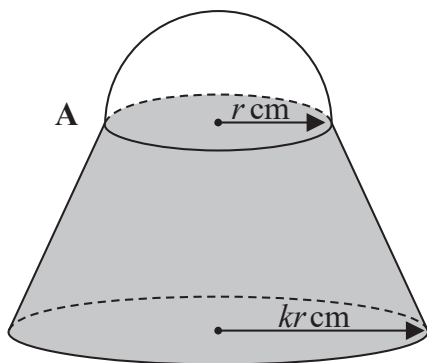


Diagram **NOT** accurately drawn



The volume of solid A is 6 times the volume of solid B.

Given that  $k > \sqrt[3]{7}$

find an expression for  $h$  in terms of  $k$  and  $r$

$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

$$\text{Large cone : } \frac{1}{3} \pi (kr)^2 \times (kh) = k^3 \left( \frac{1}{3} \pi r^2 h \right)$$

$$\text{Small cone : } \frac{1}{3} \pi r^2 h$$

$$\text{Frustum : } (k^3 - 1) \frac{1}{3} \pi r^2 h$$

$$\text{Sphere : } \frac{4}{3} \pi r^3 \quad \text{hemispheres : } \frac{2}{3} \pi r^3$$

$$V_A = (k^3 - 1) \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3$$

$$V_B = \frac{2}{3} \pi r^3 + \frac{1}{3} \pi r^2 h$$

$$V_A = 6V_B$$

$$\Rightarrow (k^3 - 1) \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 = 6 \left[ \frac{2}{3} \pi r^3 + \frac{1}{3} \pi r^2 h \right]$$

$$(k^3 - 1)h + 2r = 12r + 6h$$

$$(k^3 - 7)h = 10r$$

$$h = \frac{10r}{k^3 - 7}$$

$$(k^3 > 7 \Rightarrow h > 0 \checkmark)$$

$$h = \frac{10r}{k^3 - 7}$$

(Total for Question 24 is 6 marks)



25  $ABCD$  is a parallelogram and  $ADM$  is a straight line.

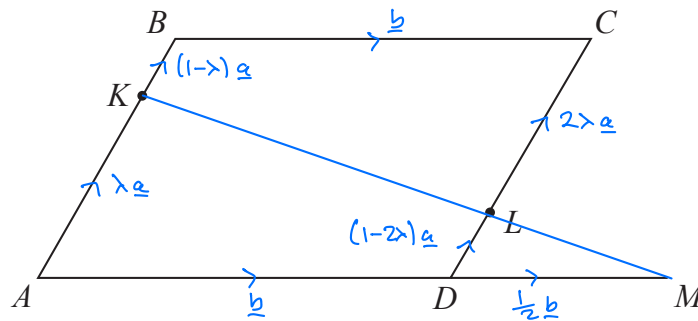


Diagram **NOT** accurately drawn

$$\vec{AB} = \mathbf{a} \quad \vec{BC} = \mathbf{b} \quad \vec{DM} = \frac{1}{2} \mathbf{b}$$

$K$  is the point on  $AB$  such that  $AK:AB = \lambda:1$   
 $L$  is the point on  $CD$  such that  $CL:CD = \mu:1$   
 $KLM$  is a straight line.

Given that  $\lambda:\mu = 1:2$

use a vector method to find the value of  $\lambda$  and the value of  $\mu$

$$\vec{AB} = \mathbf{a} \quad AK:AB = \lambda:1 \Rightarrow \vec{AK} = \lambda \vec{AB} = \lambda \mathbf{a}$$

$$\vec{CD} = \vec{BA} = -\mathbf{a} \quad (\text{ABCD is parallelogram})$$

$$CL:CD = \mu:1 \Rightarrow \vec{CL} = \mu \vec{CD}$$

$$\lambda:\mu = 1:2 \Rightarrow \mu = 2\lambda \Rightarrow \vec{CL} = 2\lambda \vec{CD} = -2\lambda \mathbf{a} \quad \text{or } \vec{LC} = 2\lambda \mathbf{a}$$

use all of this to label diagram

$$KLM \text{ is straight line} \Rightarrow \vec{KL} = x \vec{KM}$$

$$\begin{aligned} \vec{KL} &= \vec{KA} + \vec{AD} + \vec{DL} \\ &= -\lambda \mathbf{a} + \mathbf{b} + (1-2\lambda) \mathbf{a} \\ &= (1-3\lambda) \mathbf{a} + \mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{KM} &= \vec{KA} + \vec{AD} \\ &= -\lambda \mathbf{a} + \frac{3}{2} \mathbf{b} \end{aligned}$$

$$\Rightarrow (1-3\lambda) \mathbf{a} + \mathbf{b} = x \left( -\lambda \mathbf{a} + \frac{3}{2} \mathbf{b} \right)$$

$$\mathbf{a}: 1-3\lambda = -\lambda x \quad \textcircled{1}$$

$$\mathbf{b}: 1 = \frac{3}{2} x \Rightarrow x = \frac{2}{3}$$

$$\text{in } \textcircled{1}: 1-3\lambda = -\frac{2}{3} \lambda$$

$$1 = \frac{7}{3} \lambda$$

$$\lambda = \frac{3}{7} \quad \mu = 2\lambda = \frac{6}{7}$$

$$\lambda = \frac{3}{7}$$

$$\mu = \frac{6}{7}$$

(Total for Question 25 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS

