Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4MA1/1HR**

Mathematics A
PAPER: 1HR
Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
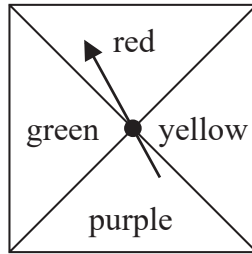
Turn over ▶

Answer ALL TWENTY FIVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Here is a biased spinner.

When the spinner is spun once, the probabilities that it lands on red or on yellow or on green are given in the table.

Colour	red	yellow	purple	green
Probability	0.25	0.2	0.35	0.2

(a) Work out the probability that the spinner lands on red or on yellow.

P (red or yellow) = 0.25 + 0.2
                  = 0.45

.......................0.45.......................

(1)

Yang is going to spin the spinner 300 times.

(b) Work out an estimate for the number of times the spinner will land on purple.

P(purple) = 1 − 0.25 − 0.2 − 0.2

          = 0.35

expected frequency = 0.35 × 300

                   = 105

.......................105.......................
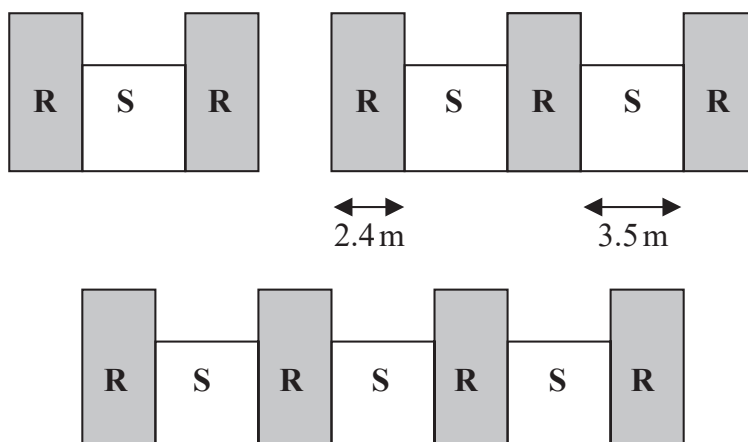
(3)

(Total for Question 1 is 4 marks)

Turn over ▶

2 In a warehouse there are two types of shelves, type **R** and type **S**.

These two types of shelves are arranged into shelving units that form a sequence of patterns.

Here are the first three terms in the sequence.

Diagram **NOT** accurately drawn

| R | S | R |

| R | S | R | S | R |

2.4 m   3.5 m

| R | S | R | S | R | S | R |

The width of each type **R** shelf is 2.4 m and the width of each type **S** shelf is 3.5 m

(a) Work out the total width of a shelving unit that has 6 type **R** shelves.

$$w = 6 \times 2.4 + 5 \times 3.5$$
$$= 31.9m$$

........................31.9........................ m

(2)

A shelving unit has n type **R** shelves.
The total width of this shelving unit is W metres.

(b) Find an expression for W in terms of n
    Give your answer in its simplest form.

$$W = n \times 2.4 + (n-1) \times 3.5$$
$$= 2.4n + 3.5n - 3.5$$
$$= 5.9n - 3.5$$

$W = $ ....5.9n - 3.5........

(2)

(Total for Question 2 is 4 marks)

3 Here are five cards.

Each card has a number written on it.

$$\boxed{15} \quad \boxed{7} \quad \boxed{-2} \quad \boxed{23} \quad \boxed{x}$$

The mean of the five numbers is 12

Work out the value of x

$$\bar{x} = \frac{\Sigma x}{n} \implies \Sigma x = n\bar{x}$$
$$= 5 \times 12$$
$$= 60$$

$$\implies 15 + 7 + (-2) + 23 + x = 60$$
$$x + 43 = 60$$
$$x = 17$$

$x = $ .....17.....

(Total for Question 3 is 3 marks)

Turn over ▶

4 The language department of a college has 180 students.
Each student studies exactly one of French, German, Italian or Spanish.

15 students study French.
45% of the students study German. $\quad 45\% = \dfrac{45}{100} = 0.45$

Express the percentage of students studying Italian or Spanish as a percentage of those studying French or German.

German : $0.45 \times 180 = 81$

French or German $= 15 + 81$
$= 96$

Italian or Spanish : $180 - 96 = 84$
as % of (French or German) : $\dfrac{84}{96} \times 100 = 87.5\%$

87.5 %

(Total for Question 4 is 4 marks)

5 (a) Expand $3c^3(c + 4)$

$c^3 \times c = c^{3+1} = c^4$

$= 3c^4 + 12c^3$

$\underline{3c^4 + 12c^3}$

(2)

(b) (i) Factorise $x^2 + 8x - 9$

$p + q = 8$ $\qquad$ $pq = -9$

$\qquad\qquad +9, -1$

$\underline{(x+9)(x-1)}$

(2)

(ii) Hence, solve $x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$x + 9 = 0$ $\quad$ or $\quad$ $x - 1 = 0$

$x = -9$ $\qquad\qquad$ $x = 1$

$\underline{-9, 1}$

(1)

(Total for Question 5 is 5 marks)

6 Show that $2\dfrac{2}{3} + 3\dfrac{3}{4} = 6\dfrac{5}{12}$

$$2\dfrac{2}{3} + 3\dfrac{3}{4} = \dfrac{8}{3} + \dfrac{15}{4}$$

$$= \dfrac{8}{3} \times \dfrac{4}{4} + \dfrac{15}{4} \times \dfrac{3}{3}$$

$$= \dfrac{32}{12} + \dfrac{45}{12}$$

$$= \dfrac{77}{12}$$

$$= 6\dfrac{5}{12} \checkmark\checkmark$$

(Total for Question 6 is 3 marks)

7 The diagram shows a solid cylinder made from iron.

Diagram **NOT** accurately drawn

3.5 cm

18 cm

$r = 9cm$
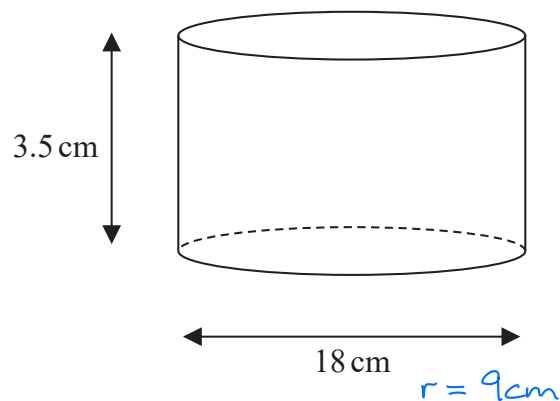
The cylinder has diameter 18 cm and height 3.5 cm
The mass of the cylinder is 7.04 kg

Work out the density of the iron.
Give your answer in g/cm^3 correct to 2 significant figures.

g/cm^3

$$V = \pi r^2 h$$
$$= \pi \times 9^2 \times 3.5$$
$$= 890.64...$$

$\Rightarrow$ need m in kg:

$$7.04 kg = 7.04 \times 1000$$
$$= 7040 g$$

$$D = \frac{M}{V}$$
$$= \frac{7040}{890.64...}$$
$$= 7.9044...$$
$$\simeq 7.9 \, g/cm^3 \quad (2sf)$$

......7.9...... g/cm^3

(Total for Question 7 is 3 marks)

P 6 8 7 8 9 A 0 9 2 8

Turn over ▶

8 Jane bought a new car for $18 000
The car depreciates in value by 15% each year. $100 - 15 = 85\% = 0.85$

Work out the value of the car at the end of 4 years.
Give your answer correct to the nearest $

$$18\,000 \times 0.85^{4} = 9396.1125$$
$$\approx \$9396 \quad (\text{nearest } \$)$$

$ ....9396............

(Total for Question 8 is 3 marks)

9 Solve the inequality $\quad 3 - 4x \leqslant 11$

$$\begin{array}{cc} -3 & -3 \end{array}$$
$$-4x \leqslant 8$$
$$\div -4 \quad \div -4 \qquad \div -ve : \leqslant \rightarrow \geqslant$$
$$x \geqslant -2$$
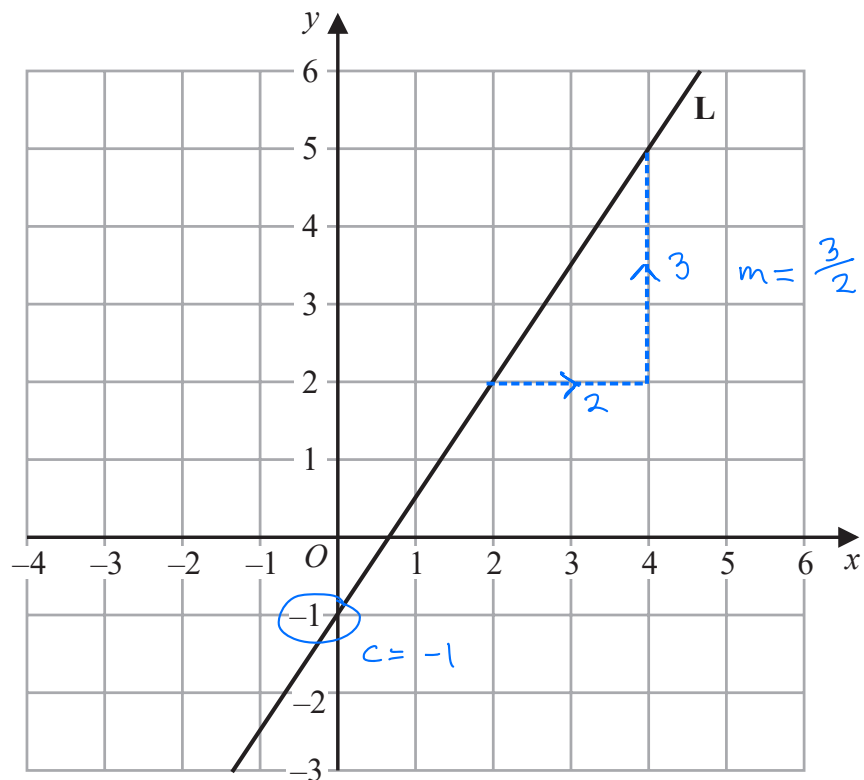
.... $x \geqslant -2$ ....

(Total for Question 9 is 2 marks)

10 Line **L** is drawn on the grid.

Find an equation for **L**

Give your answer in the form $y = mx + c$

$$y = \frac{3}{2}x - 1$$

.... $y = \frac{3}{2}x - 1$

(Total for Question 10 is 3 marks)

11 The diagram shows a quadrilateral *ABCD*
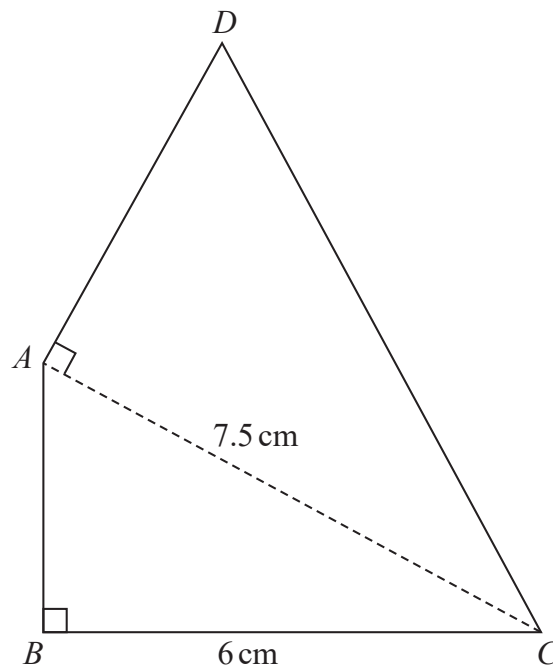
In the diagram, *ABC* and *DAC* are right-angled triangles.

$BC = 6\,\text{cm}$ $AC = 7.5\,\text{cm}$

The area of quadrilateral *ABCD* is $31.5\,\text{cm}^2$

Work out the length of *AD*

Pythagoras: $AB^2 + 6^2 = 7.5^2$

$$AB^2 = 7.5^2 - 6^2$$

$$= 20.25$$

$$AB = \sqrt{20.25}$$

$$= 4.5$$

Area $(\triangle ABC) = \frac{1}{2} \times 6 \times 4.5$

$$= 13.5\,\text{cm}^2$$

Area $(\triangle ACD) = $ Area (parallelogram) $-$ Area $(\triangle ABC)$

$$= 31.5 - 13.5$$

$$= 18\,\text{cm}^2$$

$\Rightarrow \frac{1}{2} \times 7.5 \times AD = 18$

$$AD = \frac{2 \times 18}{7.5} = 4.8\,\text{cm}$$

.... 4.8 .... cm

(Total for Question 11 is 6 marks)

P 6 8 7 8 9 A 0 1 3 2 8

12 $P = 3^3 \times 5^2 \times 7$

$Q = 3^2 \times 5 \times 7^2$

(a) Write down the highest common factor (HCF) of P and Q
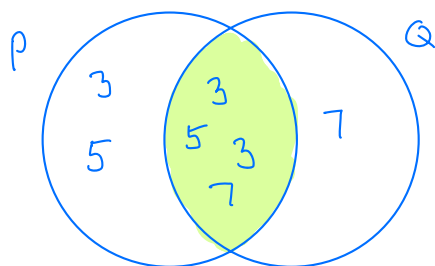
HCF $= 3 \times 3 \times 5 \times 7$

$= 315$

.......................315.......................

(1)

$P = 3^3 \times 5^2 \times 7$

$Q = 3^2 \times 5 \times 7^2$

(b) Work out the value of $P^3 \times Q$

Give your answer in the form $3^x \times 5^y \times 7^z$ where x, y and z are positive integers.

$$P^3 \times Q = (3^3 \times 5^2 \times 7)^3 \times 3^2 \times 5 \times 7^2$$

$$= 3^{3 \times 3} \times 5^{2 \times 3} \times 7^3 \times 3^2 \times 5 \times 7^2$$

$$= 3^9 \times 5^6 \times 7^3 \times 3^2 \times 5 \times 7^2$$

$$= 3^{9+2} \times 5^{6+1} \times 7^{3+2}$$

$$= 3^{11} \times 5^7 \times 7^5$$

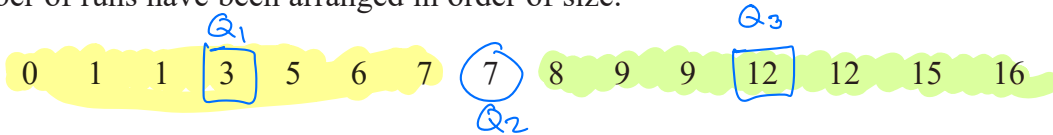..............$3^{11} \times 5^7 \times 7^5$..............

(2)

(Total for Question 12 is 3 marks)

13 Here is the number of runs scored by a baseball team in each of its 15 games this season.

The number of runs have been arranged in order of size.

Q_1

Q_3

0   1   1   3   5   6   7   7   8   9   9   12   12   15   16

Q_2

Work out the interquartile range of the number of runs.

data already in order

IQR = 12 - 3

= 9

9

(Total for Question 13 is 2 marks)

Turn over ▶

14 Solve the simultaneous equations

$$3x - 5y = 25 \quad ①$$
$$4x + 3y = 14 \quad ②$$

Show clear algebraic working.

$4 \times ① : \quad 12x - 20y = 100 \quad ③$

$3 \times ② : \quad 12x + 9y = 42 \quad ④$

$③ - ④ \quad -29y = 58$

$\qquad y = -2$

in $① : \quad 3x + 10 = 25$

$\qquad 3x = 15$

$\qquad x = 5$

$x = \underline{\quad 5 \quad}$

$y = \underline{\quad -2 \quad}$

(Total for Question 14 is 4 marks)

15
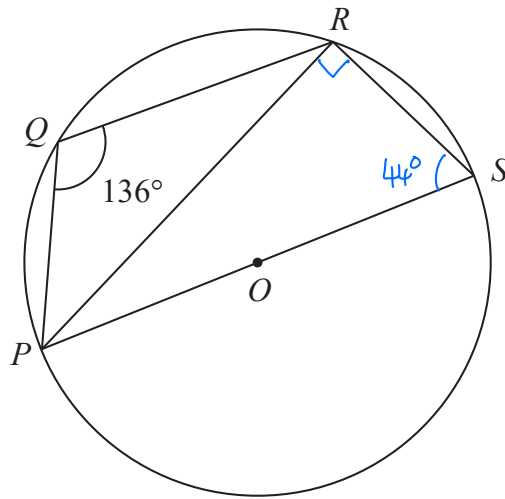
Diagram **NOT** accurately drawn

P, Q, R and S are points on a circle with centre O

PS is a diameter of the circle.

Angle $PQR = 136°$

Work out the size of angle RPS

$\angle PRS = 90°$ angle in semicircle $= 90°$

$\angle PSR = 180 - 136$ opposite angles in cyclic quadrilateral
                                  add to $180°$

      $= 44°$

$\angle RPS = 180 - 90 - 44$ angles in triangle add to $180°$

      $= 46°$

46   °

(Total for Question 15 is 3 marks)

16 (a) Expand and simplify $(3x - 1)(x + 2)(3x + 1)$

$= (3x^2 + 6x - x - 2)(3x + 1)$

$= (3x^2 + 5x - 2)(3x + 1)$

$= 9x^3 + 3x^2 + 15x^2 + 5x - 6x - 2$

$= 9x^3 + 18x^2 - x - 2$

$\underline{9x^3 + 18x^2 - x - 2}$

(3)

(b) Simplify fully $\left(\dfrac{2x^5}{8xy^2}\right)^{-2}$

$\dfrac{2 \times x \times x \times x \times x \times x}{2 \times 4 \times x \times y \times y}$

$\left(\dfrac{x^4}{4y^2}\right)^{-2}$

$= \dfrac{x^4}{4y^2}$

$= \left(\dfrac{4y^2}{x^4}\right)^2$

$4^2 = 16$

$(y^2)^2 = y^{2 \times 2}$

$(x^4)^2 = x^{4 \times 2}$

$= y^4$

$= x^8$

$= \dfrac{16 y^4}{x^8}$

$\underline{\dfrac{16y^4}{x^8}}$

(3)

(Total for Question 16 is 6 marks)

17 Here is a parallelogram *PQRS*, in which angle *SPQ* is acute.

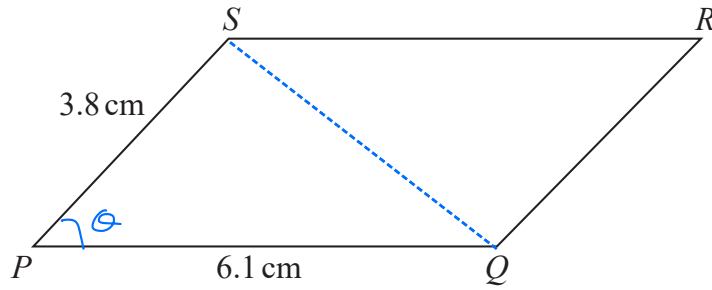S   R

3.8 cm

θ

P   6.1 cm   Q

Diagram **NOT**
accurately drawn

$PQ = 6.1\,\text{cm}$ $PS = 3.8\,\text{cm}$

The area of the parallelogram is $18\,\text{cm}^2$

Area $(\triangle PSQ) = \frac{1}{2} \times 18$

$= 9\,\text{cm}^2$

Work out the length of *QS*
Give your answer correct to 3 significant figures.

Area $(\triangle PSQ) = \frac{1}{2} \times 6.1 \times 3.8 \times \sin\theta = 9$

$$\sin\theta = \frac{9 \times 2}{6.1 \times 3.8} = \frac{900}{1159}$$

$$\theta = \sin^{-1}\left(\frac{900}{1159}\right)$$

$$= 50.944°...$$

$$SQ^2 = 3.8^2 + 6.1^2 - 2 \times 3.8 \times 6.1 \times \cos(50.944...)$$

$$= 22.43...$$

$$SQ = \sqrt{22.43...}$$

$$= 4.737...$$

$$\simeq 4.74\,\text{cm}$$

4.74 .... cm

(Total for Question 17 is 5 marks)

Turn over ▶

DO NOT WRITE IN THIS AREA   DO NOT WRITE IN THIS AREA   DO NOT WRITE IN THIS AREA

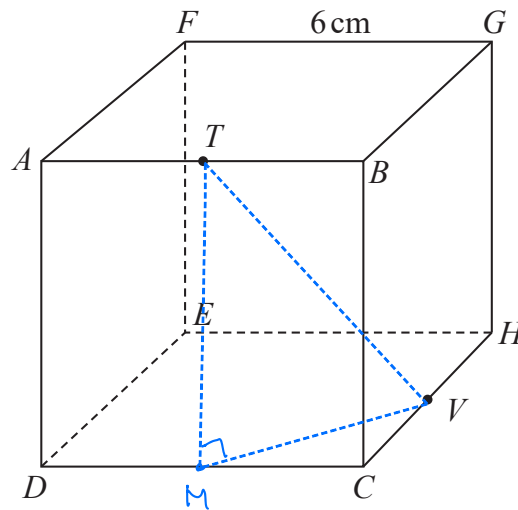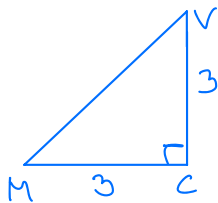**18** The diagram shows a cube *ABCDEFGH* with sides of length 6 cm.

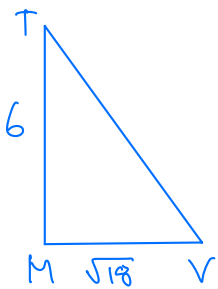Diagram **NOT** accurately drawn

T is the midpoint of *AB* and *V* is the midpoint of *CH*

Work out the distance from *T* to *V* in a straight line through the cube.
Give your answer in the form $\sqrt{a}$ cm where a is an integer.

$$MV^2 = 3^2 + 3^2$$
$$MV = \sqrt{18}$$

$$TV^2 = 6^2 + (\sqrt{18})^2$$
$$= 54$$
$$TV = \sqrt{54} \text{ cm}$$
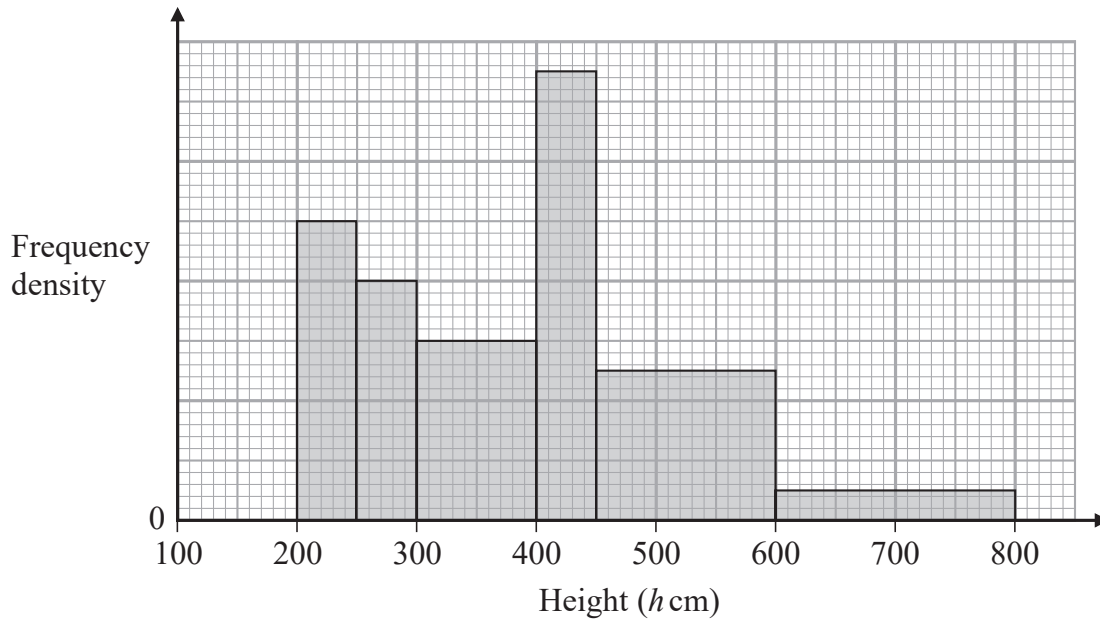
$\sqrt{54}$ cm

(Total for Question 18 is 4 marks)

19 The histogram gives information about the height, h cm, of each tree in part of a forest.

There are no trees for which $h \leqslant 200$ and for which $h > 800$

The number of trees for which $300 < h \leqslant 400$ is 8 fewer than the number of trees for which $400 < h \leqslant 500$

Work out an estimate for the number of trees in this part of the forest that have a height greater than 500 cm.

$300 < h \leqslant 400$: 6 large squares

$400 < h \leqslant 500$: $7.5 + 2.5 = 10$ large squares

difference $= 4$ large squares

$\Rightarrow$ 4 large squares represents $f = 8$

1 large square : $f = 2$

> 500 cm : $5 + 1 + 1 = 7$ large squares

$f = 2 \times 7 = 14$

$\longrightarrow$ 14 trees

.... 14 ....

(Total for Question 19 is 3 marks)

20 The diagram shows two similar metal statues.
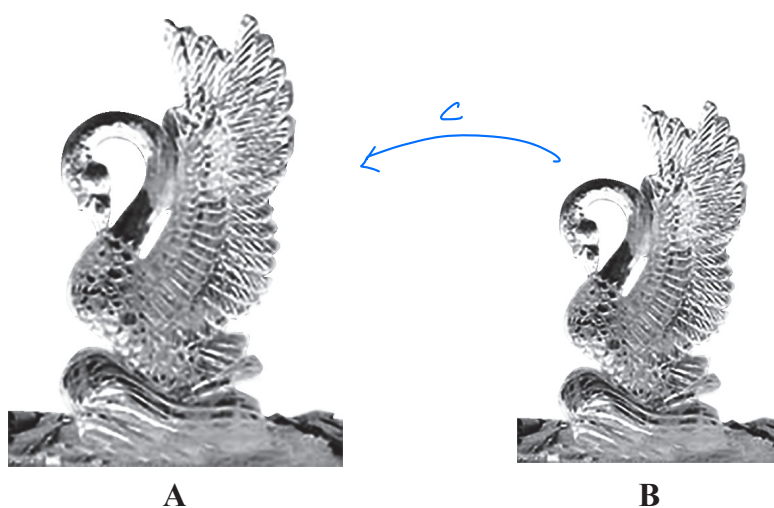
Diagram **NOT** accurately drawn

A   B

The volume of statue **B** is 20% less than the volume of statue **A** $100 - 20 = 80\% = 0.8$

The surface area of statue **B** is $k\%$ less than the surface area of statue **A**

Work out the value of k
Give your answer correct to 3 significant figures.

$$V_B = 0.8\, V_A$$

let length scale factor $= c$

$$c^3 = 0.8 = \frac{4}{5}$$

$$V_B = c^3\, V_A$$

$$SA_B = c^2\, SA_A$$

$$= (c^3)^{2/3}\, SA_A$$

$$= 0.8^{2/3}\, SA_A$$

$$SA_B = 0.86177\ldots\, SA_A$$

$\longrightarrow$ ie $SA_B = 86.177\ldots\% \; SA_A$

$\simeq 86.2\% \; SA_A$

$\longrightarrow 13.8\%$ less

$k = 13.8$

$k =$ 13.8

(Total for Question 20 is 4 marks)

21 Express $\dfrac{3+\sqrt{8}}{\left(\sqrt{2}-1\right)^2}$ in the form $p + \sqrt{q}$ where p and q are integers.

Show each stage of your working clearly.

$(\sqrt{2}-1)^2 = (\sqrt{2}-1)(\sqrt{2}-1)$

$= 2 - \sqrt{2} - \sqrt{2} + 1$

$= 3 - 2\sqrt{2}$ $\qquad 2\sqrt{2} = \sqrt{2^2 \times 2} = \sqrt{8}$

$= 3 - \sqrt{8}$

$\dfrac{3+\sqrt{8}}{(\sqrt{2}-1)^2} = \dfrac{3+\sqrt{8}}{3-\sqrt{8}} \times \dfrac{3+\sqrt{8}}{3+\sqrt{8}}$

$= \dfrac{9 + 3\sqrt{8} + 3\sqrt{8} + 8}{9 - 8}$

$= 17 + 6\sqrt{8}$ $\qquad 6\sqrt{8} = \sqrt{6^2 \times 8}$

$= 17 + \sqrt{288}$ $\qquad\qquad = \sqrt{288}$

$\underline{17 + \sqrt{288}}$

(Total for Question 21 is 4 marks)

Turn over for Question 22

22 The diagram shows a sketch of part of the curve with equation $y = x^2 - \dfrac{p}{x}$ where p is a positive constant.
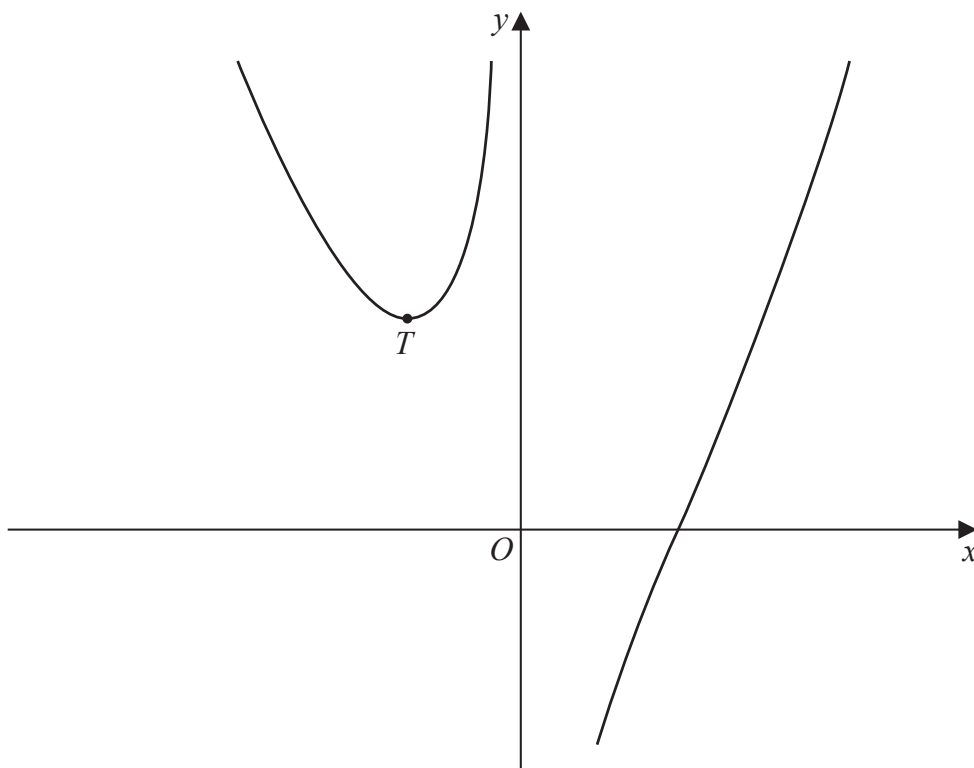

Diagram **NOT** accurately drawn

For all values of p, the curve has exactly one turning point and this turning point is a minimum shown as the point T in the sketch.

For the curve where the x coordinate of T is -3

(a) find the value of p

at T, $\dfrac{dy}{dx} = 0$

$y = x^2 - px^{-1}$

$\dfrac{dy}{dx} = 2x - p(-x^{-2})$

$= 2x + \dfrac{p}{x^2}$

$x = -3, \dfrac{dy}{dx} = 0 \implies -6 + \dfrac{p}{9} = 0$

$-54 + p = 0$

$p = 54$

$p = \underline{\ \ 54\ \ }$

(4)

The line with equation $y = k$ is a tangent to the curve with equation $y = x^2 - \dfrac{16}{x}$

(b) Find the value of k

$y = k$ is horizontal $\Rightarrow$ must be tangent
to turning point : $\dfrac{dy}{dx} = 0$
$k = y$-value of turning point

$y = x^2 - 16x^{-1}$

$\dfrac{dy}{dx} = 2x - 16(-x^{-2})$

$\quad = 2x + \dfrac{16}{x^2}$

$2x + \dfrac{16}{x^2} = 0$

$2x^3 + 16 = 0$

$x^3 = -8$

$x = -2$: $y = (-2)^2 - \dfrac{16}{-2}$

$\quad = 4 + 8$

$\quad = 12$

$\Rightarrow k = 12$

$k =$ ....12........

(3)

(Total for Question 22 is 7 marks)

Turn over for Question 23

23 (a) Express $2x^2 - 12x + 3$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

$$2x^2 - 12x + 3 = 2[x^2 - 6x] + 3$$
$$= 2[(x-3)^2 - 9] + 3$$
$$= 2(x-3)^2 - 18 + 3$$
$$= 2(x-3)^2 - 15$$

$2(x-3)^2 - 15$

(3)

The curve **C** has equation $y = 2(x + 4)^2 - 12(x + 4) + 3$

The point M is the minimum point on **C**

(b) Find the coordinates of M

This is $2x^2 - 12x + 3$ with x replaced by $(x+4)$

$$2x^2 - 12x + 3 = 2(x-3)^2 - 15$$
$$\Rightarrow 2(x+4)^2 - 12(x+4) + 3 = 2[(x+4)-3]^2 - 15$$
$$= 2(x+1)^2 - 15$$

which has minimum at $(-1, -15)$

(-1 , -15)

(2)

(Total for Question 23 is 5 marks)

24 Elliot has x counters.

Each counter has one red face and one green face.

Elliot spreads all the counters out on a table and sees that the number of counters showing a red face is 5

Elliot then picks at random one of the counters and turns the counter over.
He then picks at random a second counter and turns the counter over.

The probability that there are still 5 counters showing a red face is $\dfrac{19}{32}$

Work out the value of x
Show clear algebraic working.

Initially, 5 red + $(x-5)$ green

Still 5 red after turning 2 over : turn over 1 red & 1 green

turn over green first : now 6 red and $(x-6)$ green
(still x overall)

$$P(G_1 R_2) = \frac{x-5}{x} \times \frac{6}{x}$$

turn over red first : now 4 red and $(x-4)$ green
(still x overall)

$$P(R_1 G_2) = \frac{5}{x} \times \frac{x-4}{x}$$

Overall : $\dfrac{x-5}{x} \times \dfrac{6}{x} + \dfrac{5}{x} \times \dfrac{x-4}{x} = \dfrac{19}{32}$

$$32\left[6(x-5) + 5(x-4)\right] = 19x^2$$

$$192x - 960 + 160x - 640 = 19x^2$$

$$19x^2 - 352x + 1600 = 0$$

$$(19x - 200)(x-8) = 0$$

x must be a whole number $\Rightarrow x = 8$

$x = $ 8

(Total for Question 24 is 5 marks)

Turn over ▶

25 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.

$$S_{10} = 4S_5$$

The 8th term of this series is 45

$$U_8 = 45$$

Find the first term of this series. $\underline{\underline{a}}$
Show clear algebraic working.

$$U_8 = a + 7d = 45 \quad ①$$

$$S_{10} = 4S_5: \quad \frac{10}{2}(2a + 9d) = 4 \times \frac{5}{2}(2a + 4d)$$

$$\Rightarrow 10a + 45d = 20a + 40d$$

$$\Rightarrow 10a - 5d = 0 \quad ②$$

$$10 \times ①: \quad 10a + 70d = 450 \quad ③$$

$$③ - ②: \quad 75d = 450$$

$$d = 6$$

$$\text{in } ①: \quad a + 42 = 45$$

$$\Rightarrow a = 3$$

$$\left(\text{Check in } ②: \ 30 - 30 = 0 \ \checkmark\checkmark\right)$$

3

(Total for Question 25 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS