Answer ALL TWENTY FOUR questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

3

1 Here are the first five terms of an arithmetic sequence.

1 5 9 13 17

(a) Find an expression, in terms of n, for the nth term of this sequence.

-3 1 5 9 13 17

-4 +4 +4 +4 +4

+4 each time : $4n + \ldots$

M1

$n = 1$: $4 \times 1 + -3 = 1$

$4n - 3$ A1

(2)

The nth term of another arithmetic sequence is $3n + 5$

(b) Find an expression, in terms of m, for the $(2m)$th term of this sequence.

Replace "n" by "2m" :

$3 \times (2m) + 5$ Then simplify

$= 6m + 5$

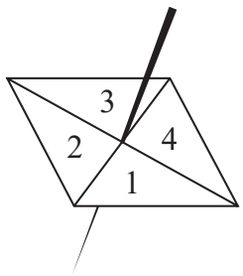
B1

$6m + 5$

(1)

(Total for Question 1 is 3 marks)

2 Here is a biased 4-sided spinner.



The table gives the probabilities that, when the spinner is spun once, it will land on 1 or it will land on 3

Number	1	2	3	4
Probability	0.26	x 0.28	0.18	x 0.28

The probability that the spinner will land on 2 is equal to the probability that the spinner will land on 4 Call this probability x (4)

Ravina is going to spin the spinner a number of times.

Ravina works out that an estimate for the number of times the spinner will land on 3 is 45

Work out an estimate for the number of times the spinner will land on 4

First find x: Probabilities add to 1

$0.26 + x + 0.18 + x = 1$ M1 (or 0.28 seen in table)

$2x = 1 - 0.44$

$= 0.56$

$x = 0.28$

(=n)

Expected frequency of A = no of spins × P(A)

3 : ef = 45 P(3) = 0.18 : $0.18 \times n = 45$

$n = \frac{45}{0.18} = 250$ M1

4 : P(4) = 0.28 ef = 250×0.28 M1

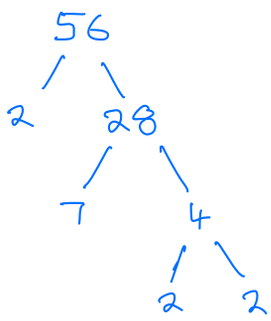
$= 70$

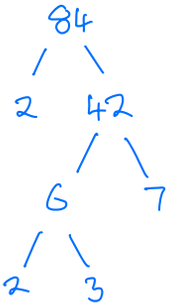
70 A1

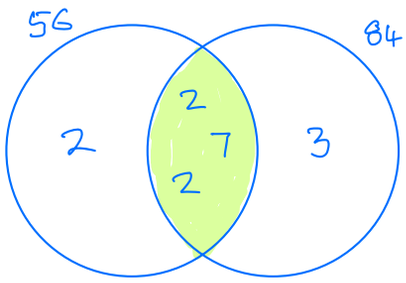
(Total for Question 2 is 4 marks)

3 (a) Find the highest common factor (HCF) of 56 and 84
Show your working clearly.

4







M1 (or list factors)

$56 = 2^3 \times 7$

$84 = 2^2 \times 3 \times 7$

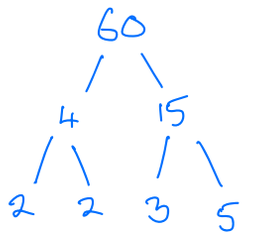
$HCF = 2^2 \times 7$

$= 28$

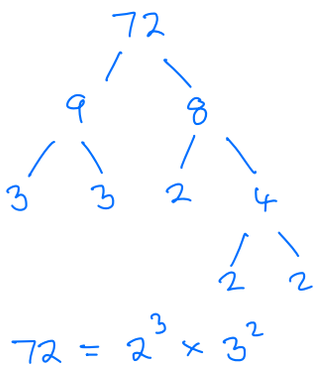
28 A1

(2)

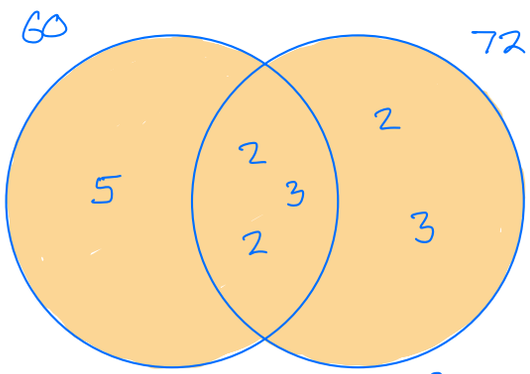
(b) Find the lowest common multiple (LCM) of 60 and 72
Show your working clearly.



$60 = 2^2 \times 3 \times 5$



$72 = 2^3 \times 3^2$



M1 (or list multiples)

$HCF = 2^3 \times 3^2 \times 5$

$= 360$

360 A1

(2)

(Total for Question 3 is 4 marks)

4 The diagram shows parts of three regular polygons, **A**, **B** and **C**, meeting at a point.

(4)

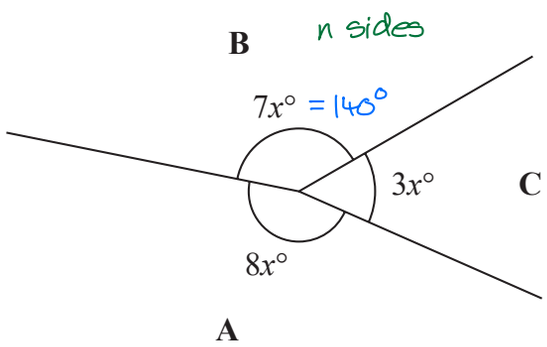


Diagram **NOT** accurately drawn

Polygon **B** has n sides.

Work out the value of n.

$7x + 3x + 8x = 360$ M1 Angles at a point add to 360°

$x = \frac{360}{18} = 20$

B : $i = 7x = 7 \times 20 = 140°$ M1

$e = 180 - 140 = 40°$

interior angle, exterior angle

$i + e = 180°$

$e = \frac{360}{n}$ $n = \frac{360}{e}$

$n = \frac{360}{40}$ M1

$= 9$ A1

$n =$ 9

(Total for Question 4 is 4 marks)

5 (a) Expand and simplify $(n-6)(n+4)$

F L I O

use "FOIL"

F: $n \times n = n^2$ I: $-6 \times n = -6n$

O: $n \times 4 = 4n$ L: $-6 \times 4 = -24$

$= n^2 + 4n - 6n - 24$ M1 (3✓)

$= n^2 - 2n - 24$ A1

5

$n^2 - 2n - 24$

(2)

(b) Solve $2x - 3 = \dfrac{3x-5}{4}$

Show clear algebraic working.

$2x - 3 = \dfrac{3x-5}{4}$

$\times 4$ $\times 4$

$4(2x-3) = 3x-5$ ← (...) essential to show whole expression $2x-3$ is $\times 4$

M1 $8x - 12 = 3x - 5$

$-3x$ $-3x$ careful: "−"

$5x - 12 = -5$

$+12$ $+12$

$5x = 7$ M1

$\div 5$ $\div 5$

$x = \dfrac{7}{5}$ A (dM1)

$x =$ $\dfrac{7}{5}$

(3)

(Total for Question 5 is 5 marks)

6 Asha bought an apartment.

The table gives information about the value of apartments, in euros, and the annual service charge band.

6

Value (x euros)	Service charge band
$x \geqslant 700\,000$	A
$600\,000 \leqslant x < 700\,000$	B
$500\,000 \leqslant x < 600\,000$	C
$400\,000 \leqslant x < 500\,000$	D
$0 < x < 400\,000$	E

634 400
610 000

In 2021, the value of Asha's apartment was 634 400 euros.

The value of Asha's apartment had increased by 4% from its value in 2020

(a) Has the annual service charge band changed for Asha's apartment?
Show your working clearly.

2021 : 634 400 had increased by 4% from 2020
in Band B

100 + 4 = 104% = 1.04 M1
÷ 100

2020 —×1.04→ 2021
2021 —÷1.04→ 2020

going backwards from 2021 to 2020 : reversing change

M1
$\frac{634\,400}{1.04}$ = 610 000 still in Band B

No, Band has not changed

A1 : No and 610 000

(3)

Pam bought a boat.

In each year after Pam bought the boat, the value of the boat depreciated by 15%

(b) Work out the total percentage by which the value of the boat had depreciated by the end of the second year after Pam bought the boat.

Depreciated : decreased by 15% 100 − 15 = 85% = 0.85
÷ 100

2 years : ×0.85² = 0.7225 = 72.25%
M1 ×100

ie boat worth 72.25% of original value after 2 years

100 − 72.25 = 27.75% decrease
M1 (or 1 − 0.7225)

A1
27.75 ..... %

(allow 27.8 or 28) (3)

(Total for Question 6 is 6 marks)

7 A cylinder is placed on the ground.

(4)

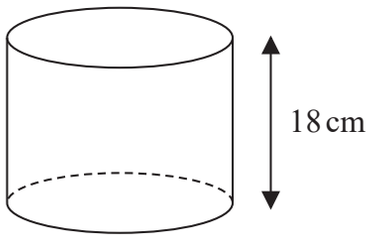
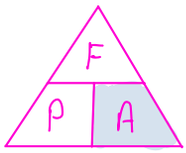


Diagram **NOT** accurately drawn

The height of the cylinder is 18 cm.

The force exerted by the cylinder on the ground is 72 newtons.
The pressure on the ground due to the cylinder is 1.4 newtons / cm^2



$A = \frac{F}{P}$

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$1.4 = \frac{72}{A}$ M1

$1.4A = 72$

$A = \frac{72}{1.4}$

$= \frac{360}{7}$ M1 Keep exact

= area of base

Work out the volume of the cylinder.
Give your answer correct to 3 significant figures.

Volume (prism)

= Area (cross-section) × height

$V = \frac{360}{7} \times 18$ M1

$\simeq 926\ cm^3$ A1 (3sf)

926 ..... cm^3

(Total for Question 7 is 4 marks)

8 (a) Write 0.000 089 in standard form.

dp here after 1st non-zero digit (2)

$= 8.9 \times 10^{-5}$

"really" 5 places to left $\Rightarrow -5$

8.9×10^{-5} B1

(1)

(b) Write 8.34×10^{4} as an ordinary number.

dp "really" 4 places to right (as +4)

$= 83\,400$

+4

83 400 B1

(1)

(Total for Question 8 is 2 marks)

9 (a) Simplify $8 \times (4t)^0$

$a^0 = 1$ (a: anything)

$= 8 \times 1 = 8$ (4)

8 B1

(1)

$x^6 \div x^{-5} = x^p$

$x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}$

(b) Find the value of p

$x^p = x^{6-(-5)}$

$= x^{11}$

$\Rightarrow p = 11$

$p =$ 11 B1

(1)

(c) Simplify fully $(2k^2m^4)^3$

$(x^a)^b = x^{a \times b}$

$2^3 = 8$ $(k^2)^3 = k^{2\times3} = k^6$ $(m^4)^3 = m^{4\times3} = m^{12}$

Don't forget to cube 2 as well as algebraic terms

B1 : 2 of $8, k^6, m^{12}$ ✓

$8k^6m^{12}$ B2

(2)

(Total for Question 9 is 4 marks)

10 Two circles, C_1 and C_2, are drawn on a centimetre grid, with a scale of 1 cm for 1 unit on each axis.

The centre of circle C_1 is at the point with coordinates (–1, 3) and the radius of C_1 is 13 cm.

The centre of circle C_2 is at the point with coordinates (7, 18) and the radius of C_2 is 6 cm.

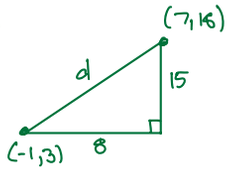
(a) Work out the distance between the centre of C_1 and the centre of C_2

$d = \sqrt{(7-(-1))^2 + (18-3)^2}$ M1

$= \sqrt{8^2 + 15^2}$ M1

$= 17$ cm A1

M1 $\sqrt{\ldots}$



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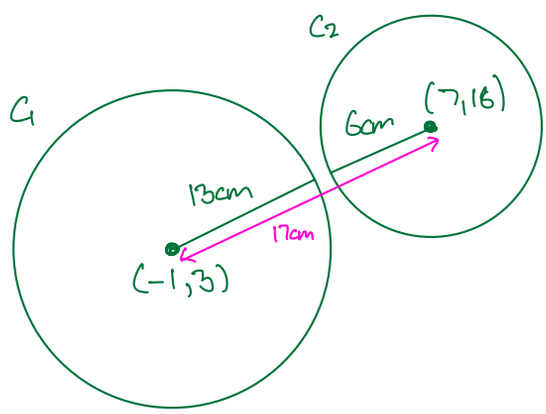


17 cm

(3)

(b) Explain why circle C_1 intersects circle C_2

$r_1 + r_2 = 13 + 6 = 19$ cm distance between centres = 17cm $< r_1 + r_2$

$\Rightarrow$ C_1 and C_2 must intersect B1 : $13 + 6 > 17$

(1)

(Total for Question 10 is 4 marks)

11 (a) Factorise $9x^2 - 4y^2$

$a^2 - b^2 = (a+b)(a-b)$

5

$= (3x)^2 - (2y)^2$

$= (3x + 2y)(3x - 2y)$ A1

M1 $(3x \pm 2y)(3x \pm 2y)$

$(3x + 2y)(3x - 2y)$

(2)

(b) Express $\frac{7}{8} - \frac{x+3}{4x}$ as a single fraction in its simplest form.

$\frac{7}{8} \times \frac{x}{x} - \frac{x+3}{4x} \times \frac{2}{2}$

Need common denominator → only multiply each fraction by extra factors needed

$= \frac{7x}{8x} - \frac{2(x+3)}{8x}$ M1

brackets essential here to show whole expression $x+3$ being $\times 2$

careful! $-2 \times 3 = -6$

$= \frac{7x - 2x - 6}{8x}$ M1

$= \frac{5x - 6}{8x}$ A1

$\frac{5x - 6}{8x}$

(3)

(Total for Question 11 is 5 marks)

12 Rudolf goes to the gym.

(4)

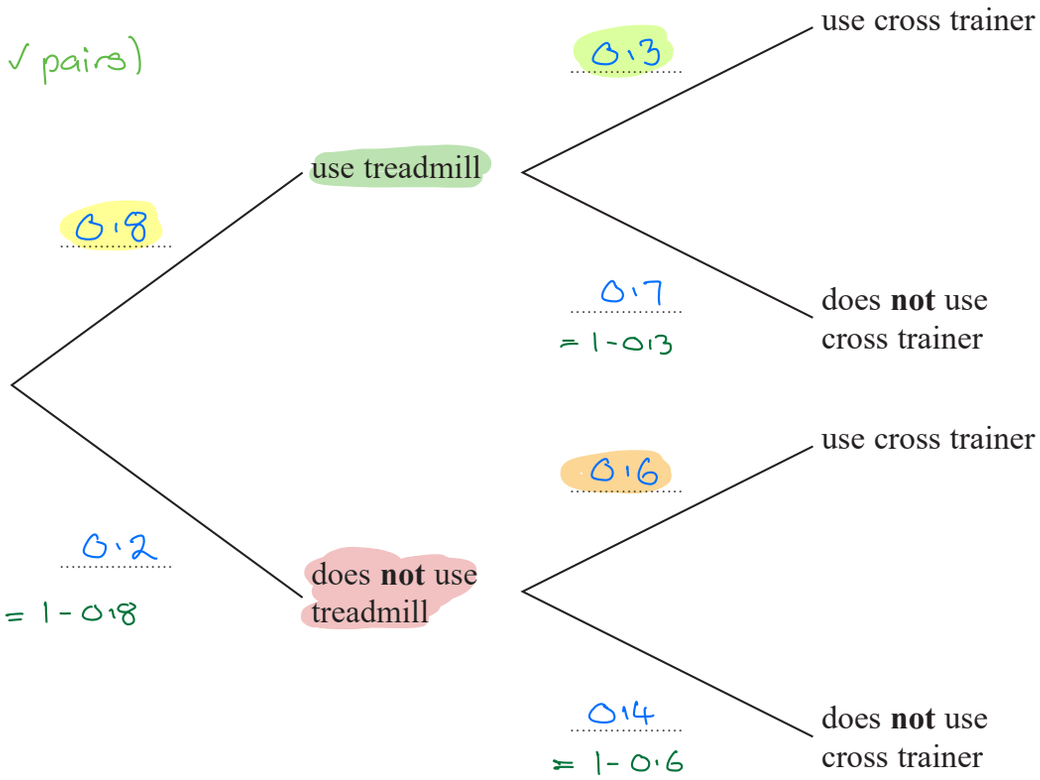
The probability that he will use the treadmill is 0.8

When he uses the treadmill, the probability that he will use the cross trainer is 0.3
When he does **not** use the treadmill, the probability that he will use the cross trainer is 0.6

(a) Complete the probability tree diagram for this information.

B2 : 3✓ pairs (0.8 & 0.2, 0.3 & 0.7, 0.6 & 0.4)

(B1 : 2✓ pairs)



(2)

(b) Work out the probability that Rudolf uses both the treadmill and the cross trainer.

P(both) = 0.8 × 0.3 M1
= 0.24 A1

↳ "AND" : ×
multiply along branches

.....

(2)

(Total for Question 12 is 4 marks)

13 Antoine is going on holiday.
He makes 3 separate payments to cover the total cost of his holiday.

The following table shows how much money Antoine pays to the holiday company.

Payment	Amount paid
Payment 1	$\frac{3}{8}$ of the total cost
Payment 2	45% of the total cost
Payment 3	\$406

5

Work out how much Antoine has to pay for Payment 2

$\frac{3}{8} + 45\% = \frac{3}{8} + 0.45$ M1

$= \frac{33}{40} = 0.825$

45% = 0.145 (÷100)

$\Rightarrow$ \$406 is $1 - \frac{33}{40} = \frac{7}{40}$ of total cost (M1)

$\frac{7}{40} C = 406 \Rightarrow C = \frac{406 \times 40}{7}$ M1 (or $\div \frac{7}{40}$)

C = total cost

$= \$2320$

2nd payment : $0.45 \times 2320 = \$1044$ M1 A1

\$ 1044

(Total for Question 13 is 5 marks)

14 The function f is defined as

(4)

$$f: x \mapsto \frac{2x}{x-6} \qquad x \neq 6$$

(a) Find f(10) Substitute 10 for x

$$f(10) = \frac{2\times 10}{10-6}$$

$$= \frac{20}{4}$$

$$= 5$$

5 B1

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1}: x \mapsto \ldots$

$y = \frac{2x}{x-6}$ Rearrange to make x the subject

$\times(x-6)$ $\times(x-6)$ Need (...) as $\times$ whole expression

M1 (either) $y(x-6) = 2x$

$xy - 6y = 2x$ Expand brackets

$-2x$ $-2x$ Collect x terms on one side...

$xy - 2x - 6y = 0$

$+6y$ $+6y$ ... and other term(s) on other side

$xy - 2x = 6y$ Take x out as factor of LHS

M1 $x(y-2) = 6y$ $\div (y-2)$

$x = \frac{6y}{y-2}$ ← can drop (...) as long as "division" line long enough & $y-2$ completely beneath

$f^{-1}: x \to \frac{6x}{x-2}$ Final inverse function must be in terms of x.

$f^{-1}: x \mapsto \frac{6x}{x-2}$ A1

(3)

(Total for Question 14 is 4 marks)

15 Abraham is going to play a computer game.
Abraham can win the game, draw the game or lose the game.

For any game that Abraham plays

the probability that he wins the game is 0.3
the probability that he draws the game is 0.5
the probability that he loses the game is 0.2

When Abraham wins a game, he scores +10 points.
When Abraham draws a game, he scores 0 points.
When Abraham loses a game, he scores −5 points.

Abraham plays 3 games and the points he scores in each of the 3 games are added together to get his total score.

Work out the probability that when he has played 3 games his total score is 0 points.

To score 0 points, must either draw all 3 games, or win 1 and lose 2

$P(DDD) = 0.5^3 = \frac{1}{8}$ $(= 0.125)$ M1

$P(WLL) = 0.3 \times 0.2^2 = 0.012$ M1

4

3 orders : WLL LWL LLW

$P(\text{win 1, lose 2}) = 3 \times 0.012 = 0.036$

$P(0 \text{ points}) = 0.125 + 0.036$ M1

$= 0.161$ A1

0.161

(Total for Question 15 is 4 marks)

16 Without using a calculator, show that $\frac{12}{\sqrt{2}-1} - \left(\sqrt{2}\right)^5 = 2\sqrt{32} + 12$

Show your working clearly.

$\frac{12}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$ M1 Rationalise denominator using $(a-b)(a+b) = a^2 - b^2$

$= \frac{12\sqrt{2} + 12}{(\sqrt{2})^2 - 1^2}$

$= 12\sqrt{2} + 12$ (3)

$(\sqrt{2})^5 = (\sqrt{2})^2 \times (\sqrt{2})^2 \times \sqrt{2}$

$= 2 \times 2 \times \sqrt{2}$

$= 4\sqrt{2}$

$\Rightarrow$ LHS $= 12\sqrt{2} + 12 - 4\sqrt{2}$

$= 12 + 8\sqrt{2}$ M1

$= 12 + 2 \times 4\sqrt{2}$ $\qquad \sqrt{a^2 \times b} = a\sqrt{b}$

$= 12 + 2\sqrt{4^2 \times 2}$

$= 12 + 2\sqrt{32}$

$=$ RHS // A1 completion

(Total for Question 16 is 3 marks)

17 A particle P moves along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds, $t \geqslant 1$, is s metres where

$$s = 4t^2 + \frac{125}{t}$$

The velocity of P at time t seconds, $t \geqslant 1$, is v m/s

Work out the distance of P from O at the instant when $v = 0$

$s = 4t^2 + 125t^{-1}$ $\quad \frac{1}{t} = t^{-1}$

$v = \frac{ds}{dt} = 8t - 125t^{-2}$ $\quad \frac{d}{dx}(x^n) = nx^{n-1}$

$= 8t - \frac{125}{t^2}$ M1 : one term $\quad t^{-2} = \frac{1}{t^2}$

A1 : both ✓

$v = 0$: $8t - \frac{125}{t^2} = 0$

5

$8t^3 - 125 = 0$

$t = \sqrt[3]{\frac{125}{8}}$ M1

$= \frac{5}{2}$ $(= 2.5\text{s})$

$t = 2.5$: $s = 4 \times 2.5^2 + \frac{125}{2.5}$ M1 (d previous M)

$= 75\text{m}$ A1

75 m

(Total for Question 17 is 5 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

18 Here is triangle ABC

5

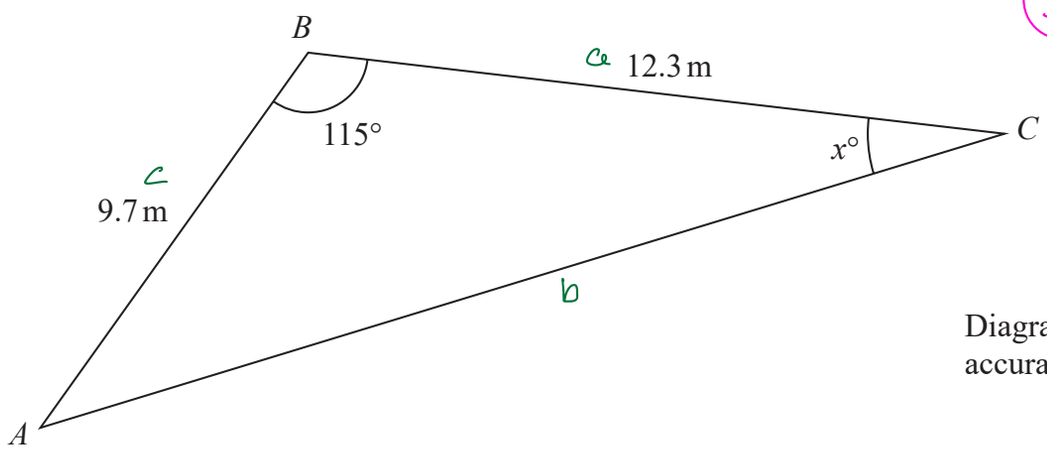


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to 3 significant figures.

First find b using cosine rule $b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 9.7^2 + 12.3^2 - 2 \times 9.7 \times 12.3 \times \cos 115°$ M1

$b = \sqrt{346.225...}$ A1

$= 18.607...$

Now find x using sine rule $\frac{\sin C}{c} = \frac{\sin B}{b}$

$\frac{\sin x}{9.7} = \frac{\sin 115°}{"18.607..."}$ M1

$x = \sin^{-1}\left(\frac{9.7 \sin 115°}{18.607...}\right)$ M1

$\simeq 28.2$ A1
(cwrt)

$x =$ .....

(Total for Question 18 is 5 marks)

19

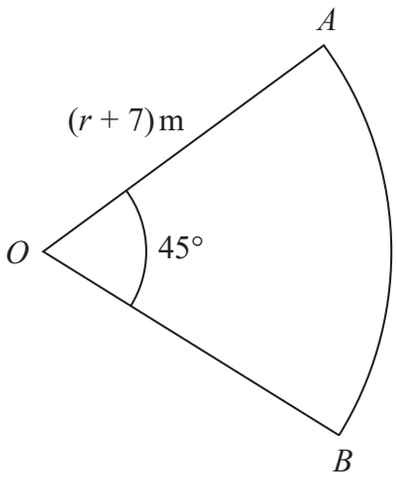


Diagram **NOT** accurately drawn

5

OAB is a sector **S** of a circle with centre *O* and radius $(r + 7)$ metres.
Angle $AOB = 45°$

A circle **C** has radius $(r - 2)$ metres.

The area of sector **S** is twice the area of circle **C**

Find the value of r
Show your working clearly.

Area Sector = 2 × area circle C

$\frac{1}{8}\frac{45}{360} \times \pi \times (r+7)^2 = 2\left[\pi \times (r-2)^2\right]$ M1 : either side M1 : equation

$(r+7)(r+7) = 16(r-2)(r-2)$ ×8 both sides

$r^2 + 7r + 7r + 49 = 16(r^2 - 2r - 2r + 4)$ Expand (..)(...) using FOIL

$r^2 + 14r + 49 = 16r^2 - 64r + 64$ $-r^2 - 14r - 49$ both sides (want $ax^2 + bx + c = 0$)

$15r^2 - 78r + 15 = 0$ ÷5 both sides

$5r^2 - 26r + 5 = 0$ A1 (dM2) either

$(5r - 1)(r - 5) = 0$ M1

$r = \frac{1}{5}$ or $r = 5$ $r \neq \frac{1}{5}$ as $r - 2$ is a length $\Rightarrow r - 2 > 0$

$\Rightarrow r = 5$ A dM2
$r = 5$ only

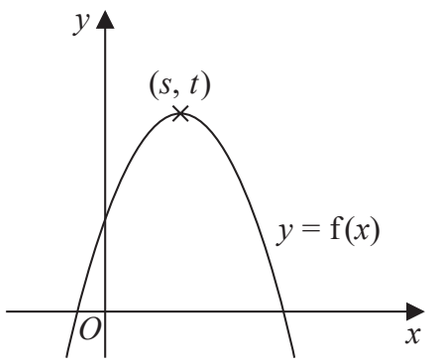
DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 19 continued.

$r =$ 5

(Total for Question 19 is 5 marks)

20 The diagram shows a sketch of part of the curve with equation $y = f(x)$



There is one maximum point on this curve.
The coordinates of this maximum point are (s, t)

Find, in terms of s and t, the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 2)$

$(x, y) \rightarrow (x+2, y)$

B1 ($s+2$, t)

(1)

(2)

(ii) $y = 3f(x)$

$(x, y) \rightarrow (x, 3y)$

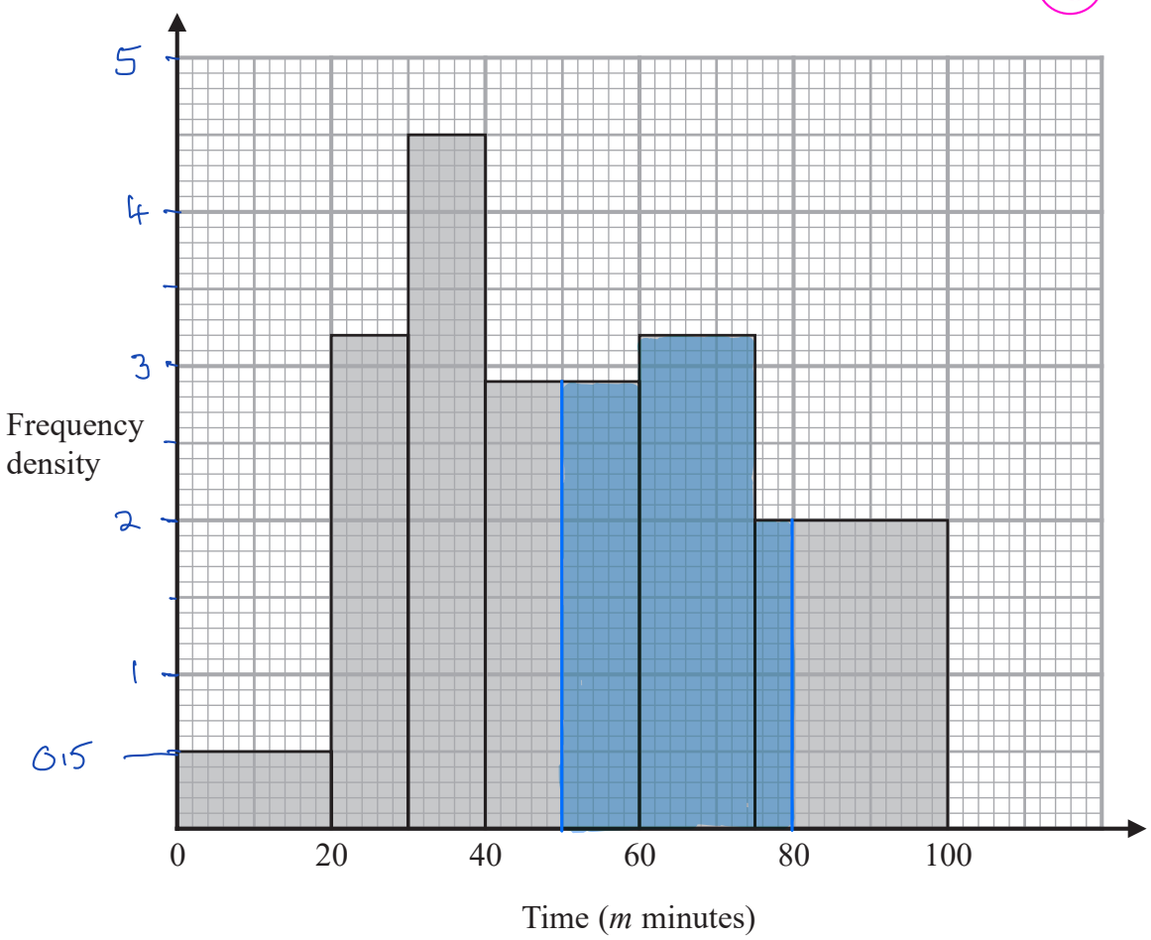
B1 (s , $3t$)

(1)

(Total for Question 20 is 2 marks)

21 The histogram shows information about the total time, m minutes, taken by each child in a school to walk to school every day for one week.

(3)



There are no children for whom $m > 100$

There are 10 children for whom $m \leqslant 20$

Area = frequency

Work out an estimate for the number of children for whom $50 < m \leqslant 80$

Area = 20 × height = 10

→ height = 0·5 M1

→ scale on y-axis

20h = 10 ⇒ h = 0·5

→ scale on y-axis

Want area between m = 50 & m = 80:

Area = 10×2·9 + 15×3·2 + 5×2 M1

= 29 + 48 + 10

= 87 A1

87

(Total for Question 21 is 3 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

22 A solid is made from a cone and a hemisphere.

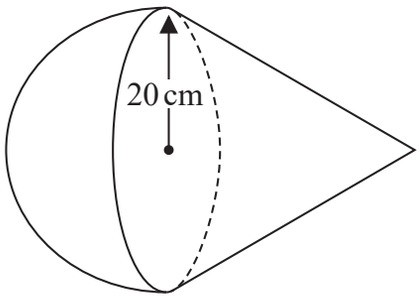


Diagram **NOT** accurately drawn

5

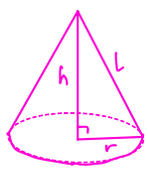
The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm.

The curved surface area of the cone is $580\pi\,\text{cm}^2$

The volume of the solid is $k\pi\,\text{cm}^3$

Work out the exact value of k

Curved SA of cone = πrl



On formula sheet

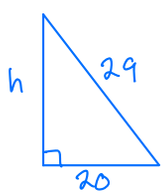
$\pi rl = 580\pi$

$(r = 20)$

$\pi \times 20 \times l = 580\pi$ M1

$l = \frac{580}{20} = 29$ M1

Need height (h) to find volume of cone: use Pythagoras:



$h^2 + 20^2 = 29^2$

$h^2 = 29^2 - 20^2$

$h = \sqrt{441}$ M1 $(\sqrt{29^2 - 20^2})$

$= 21$

$V_{sphere} = \frac{4}{3}\pi r^3$

$V_{cone} = \frac{1}{3}\pi r^2 h$

} formula sheet

$V_{solid} = V_{hemisphere} + V_{cone}$

$V = \frac{1}{2} \times \frac{4}{3}\pi \times 20^3 + \frac{1}{3} \times \pi \times 20^2 \times 21$

M1: complete method

$= \left(\frac{2}{3} \times 20^3 + \frac{1}{3} \times 20^2 \times 21\right)\pi$

$= \frac{24400}{3}\pi = k\pi$

$\Rightarrow k = \frac{24400}{3}$ A1 (exact answer required, so $8133\frac{1}{3}$ ✓ 8133.3 ✓ 8133.3 ✗)

$k = \frac{24400}{3}$

(Total for Question 22 is 5 marks)

23 A polygon has n sides, where $n > 5$

When arranged in order of size, starting with the largest number, the sizes of the interior angles of the polygon, in degrees, are the terms of an arithmetic sequence.

Here are the first five terms of this sequence.

177 175 173 171 169

Find the value of n
Show clear algebraic working.

(6)

AS : $a = 177$ $d = -2$ (M1) n terms

S_n for arithmetic series

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{n}{2}[2\times 177 + (n-1)\times(-2)]$

Sum of interior angles for n-sided polygon

$\Sigma i = (n-2)\times 180$

$= 180(n-2)$

M1 : ✓ substitution into S_n
M1 : equation
(dM2)

$\Rightarrow n(354 - 2n + 2) = 360(n-2)$

$356n - 2n^2 = 360n - 720$

$2n^2 + 4n - 720 = 0$

$n^2 + 2n - 360 = 0$ A1 (oe)

$360 = 18\times 20$
$\rightarrow +20, -18$

$(n+20)(n-18) = 0$ M1 (dM2)

$n = -20$ or $n = 18$ $n > 0 \Rightarrow$ $n = 18$ only

A (dM3) 18 only

Question 23 continued

$n =$ 18

(Total for Question 23 is 6 marks)

Turn over for Question 24

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

24 Express each of a, b and c in terms of q so that

(4)

$$q + 12x - qx^2$$

can be written as $a - b(x - c)^2$

Completing the square:

$x^2 + 2kx = (x+k)^2 - k^2$

$q + 12x - qx^2 = -q\left[x^2 - \frac{12}{q}x\right] + q$ M1

Complete square on $[\ldots]$: $2k = \frac{-12}{q}$: $k = \frac{-6}{q}$, $k^2 = \frac{36}{q^2}$

$= -q\left[\left(x - \frac{6}{q}\right)^2 - \frac{36}{q^2}\right] + q$ M1

$= -q\left(x - \frac{6}{q}\right)^2 + \frac{36}{q} + q$ M1 (oe)

Multiply out $[\ldots]$

$a - b(x-c)^2 = -b(x-c)^2 + a$

$\Rightarrow a = \frac{36}{q} + q = \frac{36 + q^2}{q}$

$b = q$

$c = \frac{6}{q}$

A1 (all 3)

$a = \frac{36}{q} + q$

$b = q$

$c = \frac{6}{q}$

(Total for Question 24 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS